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In-plane failure surfaces for masonry with joints of finite thickness estimated by a Method of Cells-type approach

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ABSTRACT

The macroscopic strength domain of in-plane loaded masonry walls is derived using an approach based on the upper bound theorem of limit analysis within the framework of homogenization theory. Following an approach similar to the Method of Cells for fiber-reinforced composites, a typical representative volume of masonry is subdivided into a few sub-cells, and a strain-rate periodic, piecewise differentiable velocity field, depending on a limited number of degrees of freedom, is defined. The ensuing approximated macroscopic failure surface is found to match with fair accuracy both available experimental data and theoretical predictions obtained by other authors with more refined numerical approaches. The proposed model is also applied to the prediction of the bearing capacity of a deep masonry beam: for any joint thickness, the criterion is found to give results as accurate as other complex numerical models, which take the heterogeneous nature of masonry into account. The model thus combines computational efficiency and accuracy.

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1. Introduction

In the last decades, there has been a growing interest in the mathematical description of the mechanical behavior of brick masonry beyond the elastic limit and up to collapse. A number of theoretical [1,2] and experimental studies [3] have been carried out, with the aim of providing reliable and efficient tools for the safety assessment of masonry structures, including monuments and buildings of historical value.

Micro-modelling is, in principle, the most refined approach to analyze masonry structures [4–8], and heterogeneous bodies in general, as the geometry and the mechanical properties of the constituent materials can be explicitly taken into account with any degree of accuracy. An intrinsic drawback of this approach is the need of modelling units and mortar joints separately. Although most authors assume joints to be interfaces of vanishing thickness, the computational effort of any micro-modelling approach is proportional to the number of bricks the structure consists of, so that its applicability is limited to small panels.

At the other extreme is macro-modelling [9–15], which does not make any distinction between units and joints, and aims at defining the mechanical properties of an equivalent homogeneous material.

Homogenization [16–24] is a fair compromise between micro- and macro-modelling. Indeed, the mechanical properties of the macroscopically equivalent material are derived from those of the constituents (brick and mortar), which can be easily obtained through cheap tests on small specimens. Once these properties have been estimated through the analysis of a Representative Volume Element (RVE), fairly rough finite element meshes can be employed to analyze large masonry buildings, assumed to consist of a homogeneous (anisotropic) material.

The major drawback of homogenization in non-linear FE computations is that a continuous interaction between meso- and macro-scale is needed. This dramatically increases the computational effort, as the field equations have to be numerically solved at each loading step, at all the integration points. For the above reasons, limit analysis combined with homogenization theory seems to be one of the most powerful and straightforward structural analysis tools to predict the ultimate bearing capacity of masonry structures in a fast and reliable way.

Different homogenization models have been recently proposed in the technical literature for the evaluation of homogenized strength domains for in-plane [20–23] and out-of-plane loaded [24–30] masonry walls. Assuming both units and joints to consist of rigid plastic materials with associated flow rule, the classical

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upper and lower bound theorems of limit analysis can be applied to any RVE to approximate the macroscopic strength domain of masonry. In particular, according to the lower bound theorem, any divergence-free plastically admissible micro-stress field, such that the stress vector is anti-periodic over the boundary of the RVE, allows a lower bound to the actual homogenized failure domain to be obtained by means of a constrained maximization problem. Conversely, the upper bound theorem deals with kinematically admissible velocity fields fulfilling suitable periodicity conditions, and allows upper bounds to the actual homogenized failure domain to be obtained by means of the constrained minimization of the internal dissipated power. In both cases, the mechanical problem is translated into a mathematical (non-)linear programming problem, with a reduced number of variables.

In the present work, the upper bound theorem of limit analysis is employed to estimate the macroscopic strength domain of in-plane loaded masonry walls, taking the finite thickness of the joints and the limited strength of both components into account. The advantage of this approach compared to existing proposals is the accuracy in the definition of the macroscopic domain, combined with the simplicity of the proposed velocity field, which depends on a very limited number of parameters.

The layout of the paper is as follows. In Section 2, the kinematic definition of the macroscopic strength domain of periodic heterogeneous media [31] is briefly recalled. The original model is presented in Section 3. In Section 3.1 a simple periodic velocity field is proposed, dividing any RVE into sub-cells according to the so-called Method of Cells (MoC) [32,33]. The ensuing limit analysis problem formulated over the RVE is detailed in Section 3.2. The main advantages of the proposed approach are summarized in Section 3.3. The model is applied in Section 4 to estimate the uniaxial off-axis compressive strength of wallettes, for which closed-form expressions are available in the literature [34,35]. In Section 5 a few models available in the literature to predict the macroscopic strength of masonry are briefly reviewed [22,26,27]: the biaxial strength domains given by the MoC at any orientation of the principal stresses to the joints are compared with those predicted by the existing models in Section 6. The failure surfaces predicted by the MoC are compared in Section 7 with the experimental results of biaxial compression tests carried out by other authors [36] on masonry panels. The implementation of the proposed criterion in a finite element code is illustrated in Section 8: the code is applied in Section 9 to predict the limit load of a deep beam. Comparisons with the predictions given by refined heterogeneous models, which accurately take the masonry texture into account, are also reported, to emphasize the accuracy of the proposed approach for any joint thickness. Finally, in Section 10 the main findings of the work are summarized and future perspectives of the research are outlined.

2. Homogenization for rigid-plastic periodic media: Kinematic definition of the macroscopic strength domain

Masonry is a composite material usually made of units bonded with mortar joints. In most cases of building practice, units and mortar are periodically arranged. Owing to periodicity, any wall Ω can be seen as the repetition of a Representative Volume Element Y (RVE, or unit cell). Y contains all the information necessary to completely describe the macroscopic behavior of Ω. If a running bond or a header bond pattern is considered (Fig. 1a), it is expedient to choose an elementary cell of rectangular shape (Fig. 1b).

To define the macroscopic (or global, or average) mechanical properties of masonry, homogenization techniques can be used both in the elastic and inelastic range, taking into account the micro-structure only at a cell level. This leads to a significant simplification of the numerical models adopted to analyze entire masonry buildings, especially in the inelastic field.

The basic idea of any homogenization procedure consists in defining averaged quantities representing the macroscopic stress Σ and the macroscopic strain rate $\boldsymbol{D}$ as follows:

$$\Sigma = \frac{1}{A}\int_Y \boldsymbol{\sigma}(\boldsymbol{y})dY, \quad \boldsymbol{D} = \frac{1}{A}\int_{\partial Y} \boldsymbol{v}(\boldsymbol{y}) \overset{s}{\otimes} \boldsymbol{n}(\boldsymbol{y})dS \tag{1}$$

where A is the area of the RVE, $\boldsymbol{y}$ is any point in Y or on its boundary ∂Y, $\boldsymbol{\sigma}$ is the microscopic (local) stress field, $\boldsymbol{v}$ is the local velocity field, $\boldsymbol{n}$ is the unit outward normal vector to ∂Y, and $\overset{s}{\otimes}$ denotes the symmetric part of the dyadic product $\boldsymbol{v} \otimes \boldsymbol{n}$. Eq. (1) applies in general to microscopic non-differentiable velocity fields.

$\boldsymbol{\sigma}$ and $\boldsymbol{v}$ must fulfil suitable periodicity conditions to match the periodicity of the heterogeneous medium:

$$\begin{cases} \boldsymbol{v} = \boldsymbol{D}\boldsymbol{y} + \tilde{\mathbf{v}}, & \tilde{\mathbf{v}} \text{ periodic on } Y \quad \text{(a)} \\ \boldsymbol{\sigma}\boldsymbol{n} & \text{anti}-\text{periodic on } \partial Y \quad \text{(b)} \end{cases} \tag{2}$$

Any velocity field fulfilling Eq. (2a) is said to be "strain-rate periodic".

The kinematic definition of the homogenized strength domain of masonry, say S^{hom}, is due to Suquet [31] and makes use of the definition of the support function of this domain, $\pi^{hom}(\boldsymbol{D})$, which reads:

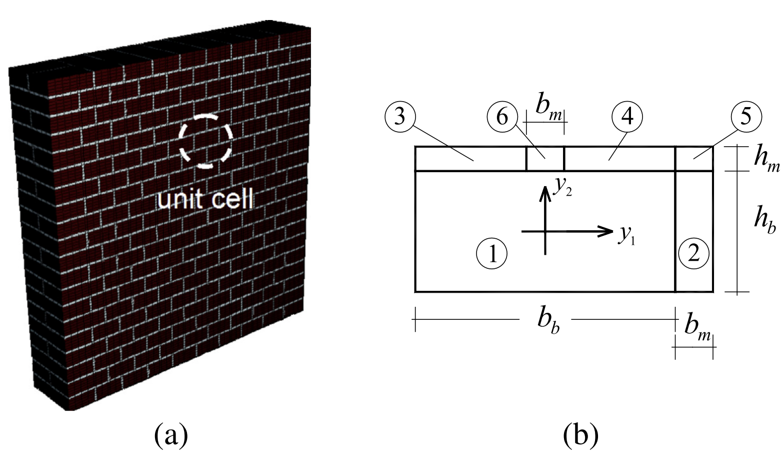

Fig. 1. (a) Running or header bond brick wall; (b) rectangular RVE and subdivision into sub-cells.

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