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Forward induction reasoning and correct beliefs ☆

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Abstract

All equilibrium concepts implicitly make a correct beliefs assumption, stating that a player believes that his opponents are correct about his first-order beliefs. In this paper we show that in many dynamic games of interest, this correct beliefs assumption may be incompatible with a very basic form of forward induction reasoning: the first two layers of extensive-form rationalizability (Pearce, 1984; Battigalli, 1997, epistemically characterized by Battigalli and Siniscalchi, 2002). Hence, forward induction reasoning naturally leads us away from equilibrium reasoning. In the second part we classify the games for which equilibrium reasoning *is* consistent with this type of forward induction reasoning, and find that this class is very small.

JEL classification: C72

Keywords: Epistemic game theory; Dynamic games; Forward induction; Common strong belief in rationality; Correct beliefs assumption; Equilibrium

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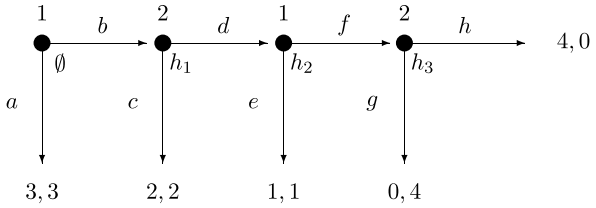

Fig. 1. Reny's game.

1. Introduction

Roughly speaking, the concepts that are used nowadays to analyze games can be divided into two categories: *equilibrium concepts* and *rationalizability concepts*. Historically, the equilibrium concepts came first, starting with the concept of Nash equilibrium (Nash, 1950, 1951), and it was only in the early eighties when rationalizability concepts systematically entered the game-theoretic picture, triggered by the pioneering work of Bernheim (1984), Pearce (1984) and Brandenburger and Dekel (1987) who developed the concept of rationalizability.

But what precisely is it that distinguishes rationalizability concepts from equilibrium concepts? To answer that question we must explicitly investigate the first-order and higher-order beliefs[1] of the players, which leads us to the field of epistemic game theory. Several papers in that literature show that equilibrium concepts make a *correct beliefs assumption*, stating that a player believes that his opponents are correct about his first-order belief, whereas rationalizability concepts do not make this assumption. For the case of Nash equilibrium this has been shown in Brandenburger and Dekel (1987, 1989), Tan and Werlang (1988), Aumann and Brandenburger (1995), Asheim (2006) and Perea (2007), which all provide epistemic characterizations of Nash equilibrium that involve, in some way, the correct beliefs assumption above. In a similar fashion, epistemic characterizations of other equilibrium concepts, like perfect equilibrium (Selten, 1975), proper equilibrium (Myerson, 1978), subgame perfect equilibrium (Selten, 1965) and sequential equilibrium (Kreps and Wilson, 1982), also rely on the correct beliefs assumption. In that light, the correct beliefs assumption may be viewed as the essential ingredient of equilibrium reasoning.

The main message of this paper is to show that within the class of dynamic games, the correct beliefs assumption, and hence equilibrium reasoning, is incompatible with a very basic form of forward induction reasoning. Therefore, in order to implement this type of forward induction reasoning we must necessarily leave the context of equilibrium reasoning. As an illustration of this fact, consider the game in Fig. 1, which is based on Fig. 3 in Reny (1992a). It is natural to assume that player 1, at the beginning of the game, believes that player 2 will not choose h at history h_3. Suppose now that player 2, at history h_1, observes that player 1 has chosen b. Since choice b can only be optimal for player 1 if he assigns a high probability to player 2 choosing h, forward induction reasoning seems to suggest that player 2, at h_1, believes that player 1 assigns a high probability to player 2 choosing h. Player 1, anticipating on this type of forward induction reasoning by player 2, therefore believes that player 2, at h_1, will be wrong about his actual first-order belief, thus violating the correct beliefs assumption.

[1] By a first-order belief we mean a belief about the opponents' choices. A second-order belief is a belief about the opponents' choices and first-order beliefs, whereas higher-order beliefs can be defined in a similar fashion.