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Instability of Belief-free Equilibria

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Abstract

Various papers have presented folk theorem results for repeated games with private monitoring that rely on belief-free equilibria. I show that these equilibria are not robust against small perturbations in the behavior of potential opponents. Specifically, I show that essentially none of the belief-free equilibria is evolutionarily stable, and that in generic games none of these equilibria is neutrally stable. Moreover, in a large family of games (which includes many public good games), the belief-free equilibria fail to satisfy even a very mild stability refinement.

JEL Classification: C73, D82. **Keywords**: Belief-free equilibrium, evolutionary stability, private monitoring, repeated Prisoner's Dilemma, communication.

1 Introduction

The theory of repeated games provides a formal framework to explore the possibility of cooperation in long-term relationships, such as collusion between firms. The various folk theorem results (e.g., Fudenberg and Maskin, 1986; Fudenberg, Levine, and Maskin, 1994) have established that efficiency can be achieved under fairly general conditions when players observe commonly shared information about past action profiles.

In many real-life situations players privately observe imperfect signals about past actions. For example, each firm in a cartel privately observes its own sales, which contain imperfect information about secret price cuts that its competitors offer to some of their customers. Formal analysis of private monitoring began with the pioneering work of Sekiguchi (1997). Since then, several papers have presented various folk theorem results that have shown that efficiency can be achieved also with private monitoring (see Kandori, 2002; Mailath and Samuelson, 2006, for surveys of this literature).

The most commonly used equilibrium in the literature on private monitoring is the *belief-free equilibrium* in which the continuation strategy of each player is a best reply to his opponent's strategy at every private history. These equilibria are called "belief-free" because a player's belief about his opponent's history is not needed to compute a best reply. Piccione (2002) and Ely and Välimäki (2002) present folk theorem results for the repeated Prisoner's Dilemma using belief-free equilibria under the assumptions that the monitoring

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