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Large roommate problem with non-transferable random utility ☆

Marcin Pęski

Department of Economics, University of Toronto

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Abstract

We analyze a large roommate problem (i.e., marriage matching in which the marriage is not restricted solely to matchings between men and women) with non-transferable utility. It is well known that while a roommate problem may not have a stable proper matching, each roommate problem does have an stable improper matching. In a random utility model with types from Dagsvik (2000) and Menzel (2015), we show that all improper stable matchings are asymptotically close to being a proper stable matching. Moreover, the distribution of types in stable matchings (proper or not) converges to the unique maximizer of an expression that is a sum of two terms: the average "welfare" of the matching and the Shannon entropy of the distribution. In the noiseless limit, when the random component of the utility is reduced to zero, the distribution of types of matched pairs converges to the outcome of the transferable utility model.

JEL classification: C78

Keywords: Matching; Random utility; Large market

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E-mail address: mpeski@gmail.com.

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1. Introduction

We analyze stable matchings in the large "roommate" model (Gale and Shapley, 1962) with random and non-transferable utility. We assume that the utility of individual i matching with individual j is drawn from a distribution

$$U_i^j \sim F\left(.|t_i, t_j, \theta_{\{i,j\}}\right) \tag{1}$$

that depends on the types t of both of individuals (for example, income, beauty, etc.), and a match-specific shock $\theta_{\{i,j\}}$. The match-specific shock is drawn from a distribution that depends on the types of the individuals. The key assumption is that distributions $F\left(.|t, t', \theta\right)$ are absolutely continuous with respect to each other at the top end of their supports. The assumption ensures that the utilities are subject to a non-trivial randomness that remains even after learning the types and match-specific shock of two individuals. A special case is the random utility specification of Dagsvik (2000):

$$U_i^j = v\left(t_i, t_j\right) + \gamma \varepsilon_i^j, \tag{2}$$

where $v\left(t_i, t_j\right)$ is the systematic part of the utility, the random shock ε_i^j is independently distributed with extreme value type I distribution, and $\gamma > 0$ is a parameter.

The roommate problem is a generalization of the classic marriage matching model in which matching is no longer restricted solely to members of two different sides of the market. In the marriage literature, the roommate problem allows to study gay marriage, including marriage markets in which the participants choose between two types of marriage. In the job-worker matching, the roommate problem allows the workers to form partnerships. The roommate problem is also closely related to the kidney exchange problem.

An example in Gale and Shapley (1962) illustrates that the roommate problem may not have a stable proper matching. For this reason, we use an approximate concept of (improper) matching where some are (badly) matched with two, instead of one, other individuals. The concept was introduced in Tan (1991) who established the existence of a (pairwise) stable (but, possibly, improper) matching. In such a matching, some individuals are improperly matched with two or more individuals, and the improper matchings form cycles. In general, the number of improperly matched individuals can be a significant fraction of the population. For instance, the replication of the example from Gale and Shapley (1962) shows that there are environments where all individuals are improperly matched in any stable matching. Alternatively, at least a third of the population must be taken out of the market in order to find a properly stable matching.[1]

Our main result establishes two asymptotic properties of *all* stable (and, possibly, improper) matchings when the population grows large and utilities are derived from the random utility model (1). First, we show that, with a probability converging to 1, the fraction of badly matched individuals in any stable matching converges to 0. It follows that all stable matchings are *asymptotically proper*. In particular, the examples similar to the replica of Gale and Shapley (1962) are not likely to appear in the large populations.

Second, we characterize the limit of the probability masses d over the pairs of types of matched individuals (so that, for instance, $d\,(t, s)$ is the mass of type t individuals matched with

[1] A k replica of the example from Gale and Shapley (1962) would have $3k$ individuals al, bl, cl for $l = 1, ..., k$ with preferences $bl >_{al} cl >_{al} \emptyset$, $cl >_{bl} al >_{bl} \emptyset$, and $al >_{cl} bl >_{cl} \emptyset$, where all other individuals are ranked below the option $\emptyset$ to stay alone. In such an environment, at least one individual from each triple must be removed in order to guarantee the existence of a stable proper matching.