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On stable and strategy-proof rules in matching markets with contracts

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Abstract

This paper studies stable and (one-sided) strategy-proof rules in many-to-one matching markets with contracts. Not assuming any kind of substitutes condition or the law of aggregate demand, we obtain the following results. First, the number of stable and strategy-proof rules is at most one. Second, the doctor-optimal stable rule, whenever it exists, is the unique candidate for a stable and strategy-proof rule. Third, a stable and strategy-proof rule, whenever it exists, is second-best optimal for doctor welfare, in that no individually rational and strategy-proof rule can dominate it. This last result is further generalized to non-wasteful and strategy-proof rules. Due to the weak assumptions, our analysis covers a broad range of markets, including cases where a (unique) stable and strategy-proof rule is not equal to the one induced by the cumulative offer process or the deferred acceptance algorithm.

JEL classification: C78; D47

Keywords: Matching with contracts; Stability; Strategy-proofness; Uniqueness; Efficiency; Irrelevance of rejected contracts

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1. Introduction

Stability and (one-sided) strategy-proofness are two leading desiderata in two-sided matching market design.[1,2] In the classic setup, it is well-known that a matching rule is stable and strategy-proof if and only if it is the one induced by the *deferred acceptance algorithm* (Gale and Shapley, 1962; Dubins and Freedman, 1981; Roth, 1982; Alcalde and Barberà, 1994). The same result holds true in the generalized *matching with contracts* model, as long as hospitals' choice functions satisfy the *substitutes* condition and the *law of aggregate demand*.[3] With these conditions, Hatfield and Milgrom (2005) verify, among many other things, that the deferred acceptance rule is stable and strategy-proof, and Sakai (2011) further shows that no other rule satisfies both desiderata. Recently, however, several real-world markets that violate the substitutes condition (and the law of aggregate demand) have been found.[4]

To cover such a broader range of markets, we study stable and strategy-proof rules with the only assumption that the choice functions on the hospital side satisfy a common mild requirement, called the *irrelevance of rejected contracts* (henceforth, IRC) condition. This rationality condition requires that if a contract is not chosen from a menu, removing it from the menu should not change the chosen set. It is logically independent of the substitutes condition and the law of aggregate demand, and is (implicitly) assumed throughout the literature.[5]

Only with this assumption, we obtain the following results. Theorems 1–2 are on the uniqueness of stable and strategy-proof rules and extend the existing results mentioned above: Theorem 1 states that the number of such rules is at most one, although there may or may not exist one without additional restrictions; and Theorem 2 establishes that the doctor-optimal stable rule is the unique candidate for a stable and strategy-proof rule, whenever it is well-defined, although it may or may not be strategy-proof without additional assumptions. Theorem 3 is on the constrained optimality of a stable and strategy-proof rule. Namely, we show that a stable and strategy-proof rule, if it exists, is never dominated in terms of doctor welfare by any other individually rational and strategy-proof rule. Furthermore, Theorem 4 shows that the same holds true even if stability is weakened to non-wastefulness in the above statement. These latter two theorems generalize similar existing results in the school choice literature (e.g., Abdulkadiroglu et al., 2009; Kesten, 2010; Kesten and Kurino, 2016).

Our approach is novel in the study of matching with contracts without the substitutes condition. The common approach in the literature is, as in Hatfield and Kojima (2010), to introduce

[1] Throughout the paper, we refer to one side of the market as doctors and the other as hospitals, whereas applications of two-sided matching theory are not restricted to medical matches.

[2] It is common in the literature to impose strategy-proofness only for the doctor side, partly because strategy-proofness for both sides is incompatible with stability on the full domain of admissible preferences (Roth, 1982). While the present study also investigates one-sided strategy-proof rules, an alternative approach is to study two-sided strategy-proofness on restricted domains (e.g., Alcalde and Barberà, 1994, Sections 4–5; Sönmez, 1999). See also Section 4.2 for further discussion on the role of the preference domain in the present study.

[3] The substitutes condition requires that if a contract is chosen from a menu, it should be also chosen when other contracts are removed from the menu. The law of aggregate demand requires that the number of chosen contracts be weakly greater when the menu enlarges in the set sense.

[4] The examples include cadet-branch matching in the U.S. Army (Sönmez, 2013; Sönmez and Switzer, 2013), affirmative actions in school choice programs and college admissions (Aygün and Turhan, 2016; Kominers and Sönmez, 2016), and lawyer-court matching in Germany (Dimakopoulos and Heller, 2014).

[5] Aygün and Sönmez (2012, 2013) point out the importance of this condition, which is implicitly assumed in Hatfield and Milgrom (2005) and Hatfield and Kojima (2010).