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Method of fundamental solutions without fictitious boundary for plane time harmonic linear elastic and viscoelastic wave problems

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ABSTRACT

This study makes the first attempt to apply a recently developed modified method of fundamental solutions (MFS) without fictitious boundary, which is named as the singular boundary method (SBM), to the solution of plane linear elastic and viscoelastic wave problems. Like the standard MFS, the SBM applies the fundamental solutions of the governing equations of interest as the basis functions. Unlike the standard MFS, the SBM, however, does not require the fictitious boundary outside physical domain to avoid the singularity of the fundamental solution and instead directly places the source points on the physical boundary coinciding with collocation points via the concept of origin intensity factors. To demonstrate the effectiveness of the SBM for plane elastic and viscoelastic wave problems, several numerical examples are given in comparison with analytical solutions, and numerical results of the MFS and the finite element method.

1. Introduction

Numerical simulation of wave propagation problems in elastic and viscoelastic media has attracted much attention in diverse scientific and engineering fields. The most commonly used numerical methods to solve such wave problems are element-based methods such as the finite element method (FEM) [1–3] and the boundary element method (BEM) [4–8]. The FEM is a domain-type method which discretizes the whole domain into a large number of small elements. And the variable of each element is described by the shape functions. Although the FEM is very successful for solving a variety of practical engineering problems, the task of mesh generation is often arduous, especially for high dimensional and complicated domain problems. In contrast to the FEM, the BEM appears as an attractive and promising numerical technique. The method has the advantage of reducing the dimensionality of the problem by one and is a high accuracy method for a wide class of problems, although the singular kernels are employed and the system matrices are non-symmetric.

Besides the BEM, the other boundary-type methods such as the method of fundamental solutions (MFS) [9–11], boundary knot method (BKM) [12], boundary particle method (BPM) [13], boundary node method (BNM) [14], regularized meshless method (RMM) [15–17], modified method of fundamental solutions (MMFS) [18], boundary distributed source method (BDS) [19], and singular boundary method (SBM) [20,21] are also developed in the past decades. These boundary-type methods are conceptually and mathematically simpler than the BEM but have less theoretical foundation. Their validity and feasibility are usually verified by numerical experiments. Among these methods, the MFS is considered as one of the first proposed and highly popular method with the merits of being truly meshfree, integration free, easy to implement and highly accurate. Its basic idea is to use the fundamental solutions of the governing equation of interest as the basis functions and place the source points on the fictitious boundary away from the physical boundary to avoid the singularity of the fundamental solution. Despite great efforts on the choice of fictitious boundary [22,23], the placement of the fictitious boundary remains tricky for complex domain problems.

In stark contrast to the MFS, the recently developed modified MFS, namely, the SBM, avoids the fictitious boundary by introducing the concept of the origin intensity factors (OIFs) which allows the source points to coincide with the collocation points on the physical domain. The regularization techniques [24–26] used in the BEM are employed to determine the OIFs for Neumann boundary conditions, and an inverse interpolation technique [20,21] is applied for the calculation of the OIFs for Dirichlet boundary conditions. The SBM maintains the merits of the MFS and has been

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applied in many engineering problems such as steady-state heat conduction [27], potential [28], plane strain elastostatic [29], acoustic [30,31], water wave [32], Stokes flow [33], and ultra-thin structural problems [34]. In this study, we make the first attempt to apply the SBM for solving plane linear elastic and viscoelastic wave problems.

The rest of the paper is organized as follows. In Section 2, the basic equations of harmonic elastic and viscoelastic problems are briefly introduced, followed by an introduction of the SBM approximation. In Section 3, we present several numerical experiments compared with the analytical solutions and the numerical solutions of the MFS and FEM to show the validity and efficiency of the SBM formulation. Finally, in Section 4, some conclusions are drawn upon the results reported in this study.

2. Basic equations and the SBM formulation

Without the loss of generality, we consider the motion of a homogeneous isotropic constant thickness plane viscoelastic medium in the frequency domain referred to the Cartesian coordinate system where x_1 and x_2 denote the horizontal and vertical coordinates, respectively. Let u_1 and u_2 represent the displacements in the x_1 and x_2-directions, $\varepsilon_{ik},\ i,k=1,2$ be the strains and $\sigma_{ik},\ i,k=1,2$ be the stresses. The strains are related to the displacement gradients by the means of

$$\varepsilon_{ik} = \frac{1}{2}\left[u_{i,k} + u_{k,i}\right], \tag{1}$$

where $()_{,i}$ denotes derivative with respect to x_i.

The constitutive relationship between strains and stresses can be expressed as [12]

$$\sigma_{ik} = \lambda\varepsilon_{ll}\delta_{ik} + 2\mu\varepsilon_{ik}, \tag{2}$$

where $\lambda = \mathrm{v}E(1+\eta j)/[(1+\mathrm{v})(1-2\mathrm{v})]$ and $\mu = E(1+\eta j)/[2(1+v)]$ are complex-valued Lamé-type elastic constants, $j=\sqrt{-1}$ the imaginary unit, E the Young's modulus, v the Poisson's ratio, η the damping ratio and δ_{ik} the Kronecker delta.

The equations of motion in the absence of body forces can be written in the following form [35,36]:

$$\frac{\partial^2 u_1(\mathbf{x})}{\partial x_1^2} + \frac{(1-v)}{2}\frac{\partial^2 u_1(\mathbf{x})}{\partial x_2^2} + \frac{(1+v)}{2}\frac{\partial^2 u_2(\mathbf{x})}{\partial x_1 \partial x_2} + \frac{\rho(1-v^2)\omega^2}{E(1+\eta j)}u_1(\mathbf{x}) = 0, \tag{3}$$

$$\frac{\partial^2 u_2(\mathbf{x})}{\partial x_2^2} + \frac{(1-v)}{2}\frac{\partial^2 u_2(\mathbf{x})}{\partial x_1^2} + \frac{(1+v)}{2}\frac{\partial^2 u_1(\mathbf{x})}{\partial x_1 \partial x_2} + \frac{\rho(1-v^2)\omega^2}{E(1+\eta j)}u_2(\mathbf{x}) = 0, \tag{4}$$

where $\mathbf{x}\in\Omega$, Ω is the problem domain, ρ the mass density, $\omega = 2\pi f$ the temporal frequency and f the frequency.

The boundary conditions for Eqs. (3) and (4) on Γ are

$$u_i = \bar{u}_i \text{ on } \Gamma_u \text{ (displacement boundary conditions)}, \tag{5}$$

$$t_i = \sigma_{ik}n_k = \bar{t}_i \text{ on } \Gamma_t \text{ (traction boundary conditions)}, \tag{6}$$

where $\Gamma = \partial\Omega = \Gamma_u \cup \Gamma_t$, n_k is the direction cosines of the outward normal on the boundary Γ.

For the multilayer domain problems, continuity conditions are imposed at the interface between two adjacent subdomains $\Omega^{(a)}$ and $\Omega^{(b)}$:

$$\begin{cases} u_i^{(a)}(\mathbf{x}) = u_i^{(b)}(\mathbf{x}) \\ t_i^{(a)}(\mathbf{x}) = -t_i^{(b)}(\mathbf{x}) \end{cases}, \quad \mathbf{x}\in\Gamma^{(a,b)}, \tag{7}$$

where $\Gamma^{(a,b)}$ is the interface between $\Omega^{(a)}$ and $\Omega^{(b)}$.

The fundamental solutions for the displacements of the systems (3) and (4) are given by

$$G_{ik}(\mathbf{x},\mathbf{s}) = \mathrm{A}\delta_{ik} - \mathrm{B}r_{,i}r_{,k}, \quad i,k = 1,2, \tag{8}$$

where

$$\mathrm{A} = \frac{1}{2\pi\omega^2\rho}\left[k_s^2K_0(jk_sr) + \frac{jK_1(jk_pr)}{r}k_p - \frac{jK_1(jk_sr)}{r}k_s\right],$$

$$\mathrm{B} = \frac{1}{2\pi\omega^2\rho}\left[k_s^2K_0(jk_sr) - k_p^2K_0(jk_pr) + 2\frac{jK_1(jk_pr)k_p}{r} - 2\frac{jK_1(jk_sr)k_s}{r}\right],$$

and $K_i(i=1,0)$ are ith order modified Bessel functions of the second kind, $k_s = \omega\sqrt{\rho/\mu}$, $k_p = \omega\sqrt{\rho/(\lambda+2\mu)}$, $r = \sqrt{(x_1-s_1)^2 + (x_2-s_2)^2}$ the distance between the source point $\mathbf{s}=(s_1,s_2)$ and the field point $\mathbf{x}=(x_1,x_2)$, $r_{,i} = (x_i - s_i)/r,\ i=1,2$. Furthermore, the fundamental solutions for the tractions are

$$T_{ik} = \lambda\left(\mathrm{A}' - \mathrm{B}' - \frac{\mathrm{B}}{r}\right)r_{,k}n_i + \mu\left[\left(\mathrm{A}' - \frac{\mathrm{B}}{r}\right)(r_{,n}\delta_{ik} + r_{,i}n_k) - \frac{2\mathrm{B}}{r}r_{,k}n_i + 2\left(-\mathrm{B}' + \frac{2\mathrm{B}}{r}\right)r_{,i}r_{,k}r_{,n}\right], \tag{9}$$

where $r_{,n} = r_{,1}n_1 + r_{,2}n_2$ and $()'$ denotes the first derivative with respect to r.

In the SBM, the displacements and tractions are approximated by the linear combinations of fundamental solutions for displacements and tractions as follows:

$$u_i(\mathbf{x}_m) = \sum_{k=1}^{2}\sum_{n=1}^{N}\alpha_{kn}G_{ik}(\mathbf{x}_m,\mathbf{s}_n), \quad \mathbf{x}_m\in\Gamma_u\cup\Omega, \tag{10}$$

$$t_i(\mathbf{x}_m) = \sum_{k=1}^{2}\sum_{n=1}^{N}\alpha_{kn}T_{ik}(\mathbf{x}_m,\mathbf{s}_n), \quad \mathbf{x}_m\in\Gamma_t, \tag{11}$$

where $\mathbf{s}_n$ is the nth source point on the boundary and α_{kn} is the unknown coefficient to be determined.

Note that the fundamental solutions become singular when $\mathbf{x}_m$ and $\mathbf{s}_m$ coincides on the physical boundary. To remove these singularities of the fundamental solutions, the concept of the origin intensity factors [21,29] was proposed in the SBM to replace the singular terms as follows

$$u_i(\mathbf{x}_m) = \sum_{k=1}^{2}\left[\sum_{n\neq m}^{N}\alpha_{kn}G_{ik}(\mathbf{x}_m,\mathbf{s}_n) + \alpha_{km}\tilde{G}_{ik}(\mathbf{x}_m,\mathbf{s}_m)\right], \quad \mathbf{x}_m\in\Gamma_u, \tag{12}$$

$$t_i(\mathbf{x}_m) = \sum_{k=1}^{2}\left[\sum_{n\neq m}^{N}\alpha_{kn}T_{ik}(\mathbf{x}_m,\mathbf{s}_n) + \alpha_{km}\tilde{T}_{ik}(\mathbf{x}_m,\mathbf{s}_m)\right], \quad \mathbf{x}_m\in\Gamma_t, \tag{13}$$

where $\tilde{G}_{ik}(\mathbf{x}_m,\mathbf{s}_m)$ and $\tilde{T}_{ik}(\mathbf{x}_m,\mathbf{s}_m)$ are origin intensity factors.

In order to obtain the origin intensity factors for traction boundary conditions, the subtracting and adding-back technique is employed as follow

$$\begin{aligned} t_i(\mathbf{x}_m) &= \sum_{k=1}^{2}\left[\sum_{n\neq m}^{N}\alpha_{kn}T_{ik}(\mathbf{x}_m,\mathbf{s}_n) + \alpha_{km}T_{ik}(\mathbf{x}_m,\mathbf{s}_m)\right] \\ &= \sum_{k=1}^{2}\left[\sum_{n\neq m}^{N}\alpha_{kn}T_{ik}(\mathbf{x}_m,\mathbf{s}_n) + \alpha_{km}T_{ik}(\mathbf{x}_m,\mathbf{s}_m) + \alpha_{km}\left(\sum_{n=1}^{N}\overline{T}_{ik}^{(e)}(\mathbf{s}_n,\mathbf{x}_m)\frac{L_n}{L_m}\right)\right] \\ &= \sum_{k=1}^{2}\left[\sum_{n\neq m}^{N}\alpha_{kn}T_{ik}(\mathbf{x}_m,\mathbf{s}_n) + \alpha_{km}\left(T_{ik}(\mathbf{x}_m,\mathbf{s}_m) + \overline{T}_{ik}^{(e)}(\mathbf{s}_m,\mathbf{x}_m)\right)\right. \\ &\quad \left. + \alpha_{km}\left(\sum_{n\neq m}^{N}\overline{T}_{ik}^{(e)}(\mathbf{s}_n,\mathbf{x}_m)\frac{L_n}{L_m}\right)\right] \end{aligned} \tag{14}$$

in which