Computers and Structures 162 (2016) 91–101

Contents lists available at ScienceDirect

Computers and Structures

journal homepage: www.elsevier.com/locate/compstruc

# An irregular lattice model to simulate crack paths in bonded granular assemblies

Christine Myra Sands

*School of Engineering, King's College, University of Aberdeen, Aberdeen AB24 3UE, United Kingdom*

ARTICLE INFO

*Article history:*
Received 2 July 2014
Accepted 22 September 2015
Available online 24 October 2015

*Keywords:*
Granular
Lattice
Inter-granule bonds
Lattice model

ABSTRACT

A 2D model of a bonded granular material is presented and its properties confirmed to be that of a brittle, isotropic elastic solid. The bond stiffnesses (axial tension/compression, shear and bending) are taken from the classical solutions to the external crack problem with two half-spaces bonded by a disc of intact material. An assembly of granules is simulated using a random array of points (representing the granule locations) with a prescribed minimum separation. The bonds are then generated by a Delaunay triangulation. This produces an isotropic array of bonds giving rise to a model material with isotropic properties. Crack growth is simulated by sequentially removing the most highly stressed bond in turn. Crack paths are then produced which are shown to agree with the predictions of linear elastic fracture mechanics, in respect of both the direction of propagation and the influence of specimen size. Some well-known problems are then simulated including: the interaction of two parallel cracks; diametrical compression of a disc; the four point bending of a beam; the influence of mortar strength on the behaviour of masonry; and a flat arch.



## 1. Introduction

This paper focuses on a model developed to simulate the response of granular assemblies to mechanically induced stress. The granules are bonded only where they are in contact, and the bond strength is significantly less than the granule strength. As such it can clearly be used to simulate the behaviour of many sedimentary rocks and a wide variety of ceramic materials, including some cement bonded blocks and many refractory products. The lattice model presented is, in effect, a sheet of three dimensional granules with all the bonds in the plane of the model. Consequently the model is currently only two dimensional; but the techniques used could easily be extended to three dimensions.

Lattice models have proved popular and have been used to simulate the behaviour of particulate materials generally [1–4]; rocks [5–8], and even a much wider range of brittle materials (e.g. concrete [9–13]) that do not have a microstructure that directly corresponds to the bonded granule idealisation.

Models of granular assemblies generally consist of:

- an assembly of granules with a lattice of bonds between (or near) the contacts;
- a prescription of bond stiffnesses;
- a prescription of bond strengths.

An assembly of granules and connecting bonds can be defined in a model in a number of different ways. Chandler [14] used a 2D hexagonal lattice of equally sized bonded spheres and created initial structural irregularities by randomly assigning some bonds to be missing. More recently, the positions, shape and orientation of granules in the model material are commonly distributed randomly and arbitrarily, but a real material can be measured and then reproduced in the model [2,5]. This approach has become more popular as computerised tomography has become more widely available. While many two dimensional models have been used successfully, three dimensional lattice models have also been developed [15,16].

Regular lattices in which all the elements have the same linear elastic properties have been shown to simulate linear elastic behaviour in the bulk material, however it has been shown [17,9] that the crack pattern produced depends very significantly on the lattice layout. A random lattice produces a more realistic result that appears to be independent of the particular lattice, but uniform material response commonly requires the properties of the elements to be adjusted to ensure that uniform strain results in uniform stress within the lattice. While that may present a problem for modelling perfectly homogeneous materials, it may not be an issue with materials that, while homogeneous at a large scale, are not so at the scale of the lattice.

Many models simulate the bonds between the nodes of the lattice (regarded as the locations of the granules in the model

*E-mail address:* c.sands@abdn.ac.uk

http://dx.doi.org/10.1016/j.compstruc.2015.09.006
0045-7949/

presented here) as springs [18,6,7,15], but beam theory is also used [1,19,20]. While most lattice models are based on linear elasticity, some have used viscous, cohesive crack, crack bridging, tension softening and other non-linear approaches [21–24] and some incorporate post-fracture compressive and frictional interaction [25–27]. While most lattice models use failure criteria based on stress, strain based systems have been used [19].

Lattice-like models using discrete element methods (DEM) have also been widely used [28,8,29–31] and although they have generally been limited to considering spherical or ellipsoidal granules, the bond stiffnesses are often similar to those used in more obviously lattice models. Discrete element methods also have the advantage that new contacts can be formed, and previously fractured bonds re-formed, that can pass shear and compression (though not tension) between granules.

Several continuum models [32–35] that can simulate crack growth in brittle materials have been developed based on a damage mechanics approach. Usually the stress and strain within the body is computed based on linear elastic theory and a threshold is set for the onset of damage. The elastic modulus is then permitted to degrade in accordance with a rule, or rules, that may be based on some mechanistic concept [32], be guided by empirical work [33], or by a widely accepted geomechanical model [34], or simply follow a realistic tension softening profile [35]. Although such models have advantages, such as the ease with which they can be incorporated into finite element analysis, they cannot capture the material behaviour at a fine scale. Lattice-particulate models, of the type presented in this paper, have an advantage in that respect.

The model presented in this paper is based on the linear elastic theory for the bending, tension and shear of two half-spaces; and bond failure is assumed to occur in pure tension. It focusses on simulating crack path development by sequentially breaking (and then removing from consideration) the most severely stressed bond. This is done one bond at a time, permitting stable crack growth to be simulated and also unstable crack growth in situations where inertial effects are negligible. The crack paths produced have been compared with recognised criteria [36] and observations [37–39]. Finally the model is used to explore qualitatively the influence of mortar strength on the performance of structural masonry. In these simulations the model is used for the purposes of comparison only and the simulations do not relate to any specific material. It follows that any numerical values provided in respect of, for example, loads or displacements are not significant.

A variety of different models have been used to simulate the response of masonry to applied load or foundation movement. Vertical loads were traditionally handled by considering the masonry as a no-tension structure and ensuring stability while avoiding local crushing of the material. More modern approaches take account of the bonding effect of the mortar joints. Pande et al. [40], developed a procedure to establish orthotropic material properties for masonry panels which was then applied that to finite element analysis modified by a smeared crack approach [41]. Alpa and Monetto [42] applied theories relating to the frictional response of arrays of microcracks and considered walls with in-plane loading, while Anthoine [43] used the theory of periodic media to establish homogenised properties. Luciano and Sacco [44] used a representative volume based on a masonry unit with adjoining mortar and considered the possible crack paths that could form and the consequent states of the masonry to determine characteristic moduli for each state. Homogenization techniques were then adopted to establish a nonlinear constitutive law for the masonry material.

A common approach for thin walls subjected to lateral loads treats masonry panels as plates [45–47], and that approach can be extended to include vertical loads [48]. Another technique [49,50] uses discrete element methods, often supported by homogenization techniques and the use of representative volumes. While these approaches have several advantages, they lack the simplicity of the model presented here.

## 2. Construction of the lattice

### 2.1. The nodes and bonds

In the model presented here, random points are generated in succession and discarded if a new point falls outside a defined area or is too close to any other existing point. In this way an irregular, but reasonably evenly distributed, set of points is created. Each of these points represents the location of the centroid of a granule and forms a node in the lattice. Each node can translate in two dimensions and rotate. A Delaunay triangulation is then performed to define which nodes are connected to a neighbouring node. Each bond connects two nodes and the granule to granule bond is assumed to exist halfway between the two connected nodes. The model, therefore, models non-spherical granules.

Fig. 1 illustrates a typical granular structure of the sort being simulated in the work presented here, overlain by the lattice structure used to simulate the granular assembly. The blue dots are nodes in the lattice and each node represents the centroid of a grain. The bonds between the grains are represented by the lattice members (shown as red lines) and the contacts between the grains are assumed to act at the mid-point of the lattice member. In the simulations presented in this paper the area of the bond connecting two grains was, on average, about 1.5% of the surface area of each of the grains that the bond connected, with a maximum of 3.7%; justifying the assumption that the contact area was small. The diameter of the grains was, on average, about 1.2% of the size of the body being simulated.

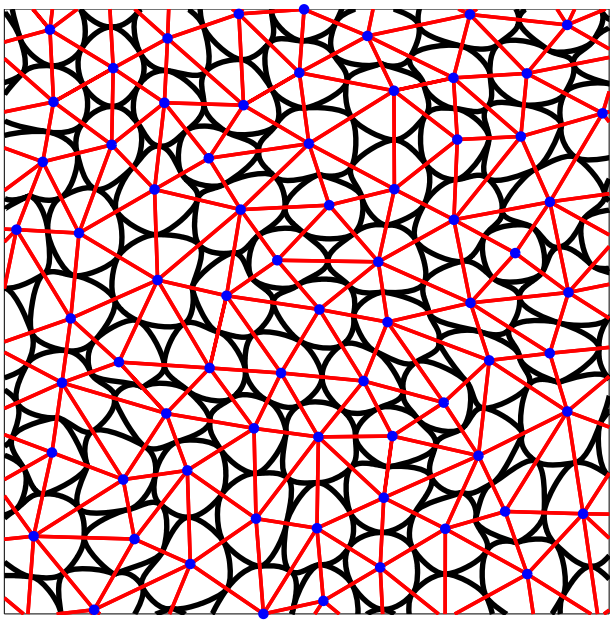

**Fig. 1.** A typical granular assembly (shown as black lines) with the structure of the model representing the assembly superimposed. The nodes that represent the grains are shown as blue dots, and the lattice members that encapsulate the bonds are shown as red lines. The nodes act at the centroid of the grains and the contacts are taken as acting at the mid-point of the lattice members that connect the nodes. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)