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Improving the accuracy of asset price bubble start and end date estimators

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ABSTRACT

Recent research has proposed using recursive right-tailed unit root tests to date the start and end of asset price bubbles. In this paper an alternative approach is proposed that utilises model-based minimum sum of squared residuals estimators combined with Bayesian Information Criterion model selection. Conditional on the presence of a bubble, the dating procedures suggested are shown to offer consistent estimation of the start and end dates of a fixed magnitude bubble, and can also be used to distinguish between different types of bubble process, i.e. a bubble that does or does not end in collapse, or a bubble that is ongoing at the end of the sample. Monte Carlo simulations show that the proposed dating approach out-performs the recursive unit root test methods for dating periods of explosive autoregressive behaviour in finite samples, particularly in terms of accurate identification of a bubble's end point. An empirical application involving Nasdaq stock prices is discussed.

1. Introduction

There is now a substantial body of research suggesting that asset price bubbles and their ensuing collapses can have a significant impact on a country's macroeconomic performance.[1] Hence, detecting the presence of an asset price bubble and the timing of its termination is of crucial importance to central banks and financial regulators, as well as investors. Of particular interest to researchers in this area are "rational bubbles" – where the real price of an asset is assumed to be equal to the present value of relevant fundamentals and a bubble component that grows in expectation at the real interest rate, and investors are assumed to have rational expectations. Under these assumptions investing in the asset can be a rational choice for investors even though its current observed price is higher than the price level that is justified by relevant fundamentals. To define a rational bubble algebraically consider the simple case of a single stock, where P_t denotes the observed real stock price, D_t denotes the observed real dividend for the stock and r denotes the real interest rate used for discounting expected future cash flows. Define the observed price as consisting of a fundamentals component and a bubble component

$$P_t = P_t^f + B_t$$

where the fundamentals component P_t^f is given by

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[1] See for example Bernanke (1995, 2013) and Greenspan (2007), Chapter 8, and the references therein, where examples of causal links between speculative bubbles, crises in banking systems and subsequent falls in aggregate demand leading to major macroeconomic recessions are discussed (including for the US stock price bubble in September/October 1929, the real estate and stock price bubbles in Japan in the late 1980s, and the US house price bubble in 2006.)

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$$P_t^f = \sum_{i=1}^{\infty} (1 + r)^{-i} E_t(D_{t+i}).$$

If the bubble component satisfies the stochastic difference equation

$$B_{t+1} = (1 + r)B_t + u_{t+1}$$

where $E_{t-i}(u_{t+1}) = 0$ for all $i \geq 0$, then a rational bubble is said to exist (cf. Diba and Grossman, 1988).

It is clear from the algebraic representation given above that in the presence of a rational bubble, since the bubble grows at an explosive rate, the observed price will be a statistically explosive process (even if the fundamentals component of the price is not statistically explosive). Recognition of this feature of rational bubbles led Diba and Grossman (1988) to propose statistical testing for the presence of a rational stock price bubble by attempting to detect explosive autoregressive behaviour in the stock price series that is not driven by similar explosive behaviour in the dividend series, using orthodox unit root tests such as the Dickey-Fuller (DF) test applied to the price and dividend series in levels and first differences. Since differencing an explosive autoregressive process does not lead to a stationary process, a rejection from the DF test for the first difference of the price and dividend series, with no rejection for the series in levels, suggests that no rational bubble exists.

The research on testing for rational bubbles by Diba and Grossman (1988) focuses on the specific case of an explosive rational bubble that does not collapse. As noted by Evans (1991) however, this type of bubble is empirically unrealistic because it implies that the asset price will perpetually grow at an explosive rate. Evans (1991) proposes a more realistic rational bubble model where the explosive bubble periodically collapses to a lower level, and the frequency of the collapses is controlled by a Bernoulli process. Using simulations, it is shown that even when the probability of collapse at each observation is extremely small so that there are just one or two collapses over the sample period considered, the use of orthodox unit root tests to detect bubbles as suggested by Diba and Grossman (1988) will tend to lead to the erroneous conclusion that a bubble is not present. This is due to the large adjustment in the price series caused by the bubble process collapsing. In effect, this creates an appearance of mean reversion, causing the series to appear to be a stationary process with no explosive behaviour.

Recognizing this weakness of orthodox DF tests, researchers have focused on developing methods for detecting asset price bubbles that are more robust to the presence of collapses in the bubble process. Initial developments in this area employed unit root tests derived from regime switching models, such as Markov-switching models (e.g. Van Norden and Vigfusson, 1998; Hall et al., 1999) and smooth transition autoregressive models (e.g. McMillan, 2006). Markov-switching models combined with Bayesian estimation techniques have also been found to be informative about the presence of bubbles that periodically collapse (e.g. Balke and Wohar, 2008). Whilst these methods have considerable advantages in the presence of periodically collapsing bubbles relative to using orthodox unit root tests, they can be computationally expensive, and the asymptotic distributions of unit root test statistics computed using these types of regime switching models are in some cases impossible to derive analytically.

Many of the more recently developed techniques for testing for bubbles retain use of DF-type tests; however, rather than applying traditional left-tailed DF tests to the price and fundamentals data in levels and differences, this research has recommended the use of *right-tailed* DF tests of the unit root null hypothesis against the alternative hypothesis of explosive autoregression applied to the relevant series in levels only. For example, see the papers by Phillips et al. (2011) (PWY), Homm and Breitung (2012), and Phillips et al. (2015) (PSY). PWY suggest constructing right-tailed DF tests recursively, and taking the supremum of this sequence of test statistics to test the unit root null hypothesis against the explosive alternative. Homm and Breitung (2012) consider a modified version of the PWY methodology, based on taking the supremum of backward recursive DF statistics. PSY recommend a statistic based on the supremum of both forward and backward recursively computed DF statistics. Simulations show that the proposed tests have very good finite sample power to detect a rational bubble, even if the bubble periodically collapses as in Evans (1991). Note also that as pointed out by PSY, a further attractive feature of the test statistics proposed in this line of research is that as well as being able to detect rational bubbles, the test statistics will have non-trivial finite sample power to detect other types of explosive bubble processes, including for example intrinsic bubbles (Froot and Obstfeld, 1991), herd behaviour (Avery and Zemsky, 1998; Abreu and Brunnermeier, 2003), and bubbles generated by time varying discount factor fundamentals (Phillips and Yu, 2011).

At least as important as being able to detect the *presence* of a bubble is the issue of being able to accurately determine the *start and end dates* of a bubble regime that is deemed to exist. Such information can be crucial ex post for reconciling the origination and termination of a bubble with other economic and financial events. Both PWY and PSY address this important issue, proposing estimators for the timing of explosive behaviour that are based on the sequences of DF recursive statistics exceeding threshold values. For example, PWY apply their approach to data on the Nasdaq composite index 1972:3–2005:6, and find evidence of explosiveness that started in 1995, predating comments made by the Federal Reserve Board Chairman, Alan Greenspan, in December 1996 on “irrational exuberance” affecting the US stock market (Greenspan, 1996).

In this paper we suggest alternative estimators of the origination and termination of a bubble period. Specifically, rather than using sequences of recursive DF statistics to date the bubble regime, we propose estimating regime change-points on the basis of model-based minimum sum of squared residuals estimators (in the spirit of, *inter alia*, Bai and Perron, 1998, and Kejriwal et al., 2013) combined with Bayesian Information Criterion (BIC) model selection. The proposed dating algorithms also allow identification of the particular form of bubble among a set of candidate bubble processes, which allow variously for a bubble that ends within the sample period, possibly with some form of collapse, or a bubble that is ongoing at the end of the sample. For a fixed magnitude bubble, we find that our BIC-based approach delivers consistent estimation of the exact bubble start and (where appropriate) end dates. Moreover, finite sample simulations suggest that, conditional on having detected the presence of a bubble using the PSY test, the new procedure offers considerably improved dating accuracy relative to the recursive DF statistic-based