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Consistent nonparametric specification tests for stochastic volatility models based on the return distribution☆

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ABSTRACT

This paper develops nonparametric specification tests for stochastic volatility models by comparing the nonparametically estimated return density and distribution functions with their parametric counterparts. Asymptotic null distributions of the tests are derived and the tests are shown to be consistent. Extensive Monte Carlo experiments are performed to study the finite sample properties of the tests. The proposed tests are applied in a number of empirical examples.

1. Introduction

In this paper we consider specification tests for a class of parametric stochastic volatility models, given by

$$\mathrm{d}Y_t = \sigma_t \mathrm{d}B_t,\ \mathrm{d}\sigma_t^2 = b(\sigma_t^2;\theta)\mathrm{d}t + a(\sigma_t^2;\theta)\mathrm{d}W_t, \tag{1}$$

where $(B_t, W_t)_{t\geq 0}$ is a bivariate standard Brownian motion process, where b and a are known functions, and where θ is an unknown parameter vector. The model is tested within a larger class of nonparametric stochastic volatility models

$$\mathrm{d}Y_t = \sigma_t \mathrm{d}B_t, \tag{2}$$

where $(\sigma_t)_{t\geq 0}$ is a stochastic process satisfying certain regularity conditions. The model (2) is nonparametric in the sense that there is no parametric structure specified for the volatility process.[1]

Model (1) is often used in financial econometrics to describe a logarithmic stock price process $(Y_t)_{t\geq 0}$, where $(\sigma_t)_{t\geq 0}$ is an unobserved spot volatility process. It includes popular models such as the Hull-White model, the Heston model and the GARCH diffusion model, which motivates the development of specification tests for this class of models. The extension of the methods developed in this paper to the case where the parametric model (1) is augmented with jumps and leverage effects will be discussed in Section 5.

Let Y be observed discretely at times $t_i = i\Delta$, $i = 0, 1, \ldots, n$. Consider the re-scaled Δ-period return sequence

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[1] Jensen and Maheu (2010) consider a Bayesian semiparametric stochastic volatility model, where the distribution of return innovations $\mathrm{d}B_t$ is assumed unknown and modeled nonparametrically. Their model is different from ours, because we test a fully parametric model against a model with a non-parametric volatility process but parametric (Gaussian) innovation distribution.

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$$X_i = \frac{1}{\sqrt{\Delta}}(Y_{t_i} - Y_{t_{i-1}}) = \frac{1}{\sqrt{\Delta}}\int_{t_{i-1}}^{t_i} \sigma_s \mathrm{d}B_s, \quad i = 1, \ldots, n. \tag{3}$$

Let $(X_i)_{i=1}^n$ having a stationary density, denoted by $q(x)$, and let $q(x;\theta)$ be its specification implied by the parametric model (1). In this paper we propose to test the specification (1) by comparing the estimated parametric return density to its nonparametrically estimated counterpart. Stated formally, we are testing

$$\mathcal{H}_0: q(x) = q_0(x) \in \{q(x;\theta), \theta \in \Theta\}, \tag{4}$$

where $\Theta \subseteq \mathbb{R}^k$ is the parameter space, and θ_0 is the true parameter under the null hypothesis: that is, it satisfies $q(x;\theta_0) = q_0(x)$.

Specification tests based on the stationary marginal return distribution have their empirical justifications — as discussed in Section 3.3 of Aït-Sahalia et al. (2010), reproducing the stationary distribution is an important aspect of structural economic modelling. The return distribution is also widely used as the basis to formulate specification tests for continuous-time diffusion processes, see e.g. Aït-Sahalia (1996) and Gao and King (2004). Admittedly, formulating the test based on $q(.;\theta)$ will limit its power in detecting certain deviations in the functions $\{b(.;\theta), a(.;\theta)\}$[2]; however, tests constructed this way would still be an important "first check" because of its empirical significance in any structural modelling. More detailed information could be obtained by defining test statistics based on the transition distribution of the observed returns. We discuss this issue in Section 8.

To formulate the test statistic, one can compare either the density functions or the cumulative distribution functions. It is known from the literature that generally speaking, density-based tests are more sensitive to local deviations, whereas distribution-based tests are more sensitive to global deviations (see e.g. Eubank and LaRiccia, 1992; Escanciano, 2009; Aït-Sahalia et al., 2009), we thus consider both in this paper.

A long-span asymptotic scheme is used in this paper. That is, we consider the asymptotics when $n \to \infty$ with fixed Δ. This is because model (1) is often used to describe price processes observed at relatively low frequencies (usually daily); intraday variation of volatility (the so-called diurnal effect) and prominent microstructure noise effects in prices observed at ultra high frequencies (see e.g. Andersen and Bollerslev, 1998) make the model unsuitable for such data. Throughout, we would need $(X_i)_{i=1}^n$ to be a stationary and ergodic sequence, and that it is β-mixing with exponentially decaying coefficients. We do not impose these properties as high level assumptions; instead, checkable sufficient conditions for these properties to hold in the parametric model are given in Appendix A.

The stochastic volatility model we consider here is essentially a (partially observed) two dimensional diffusion process, so our test is related to the vast literature on nonparametric tests for diffusion models, such as Aït-Sahalia (1996), Hong and Li (2005), Corradi and Swanson (2005), Li (2007), Chen et al. (2008), Aït-Sahalia et al. (2010), Kristensen (2011) and Aït-Sahalia and Park (2012), among others. However, the unobservability of the volatility process in (1) makes the aforementioned research not directly applicable. Corradi and Swanson (2011), henceforth CS11, consider a conditional distribution based nonparametric test for stochastic volatility models, where the authors assume the observed series to be strictly stationary. In our model we assume the observed return series (first difference of the observed series) to be stationary and we allow the observed series to exhibit unit-root type dynamics. Although CS11's method can be applied to a range of interest rate stochastic volatility models, where mean-reversion is often observed, it does not cover the well known Heston model and GARCH diffusion model, which are used widely in the option pricing literature to describe the evolution of equity prices. Zu (2015) analyzes an alternative approach to a similar testing problem, by comparing the nonparametric kernel deconvolution estimator of the volatility density with its parametric counterpart.

The contributions of the paper are threefold. First, although it appears that CS11's work can be adapted to the equity price stochastic volatility models easily, no formal work has been done so far (according to our knowledge); besides, our the marginal distribution based tests are by no means a direct adaptation of CS11's test and the asymptotic theory is different: CS11 use conditional empirical process techniques while we use a central limit theorem of U statistics for our density based tests and use empirical process techniques for our distribution function based test. Second, instead of imposing high level assumptions that the observed series is stationary and strong mixing as in CS11, we give checkable conditions in our paper for the necessary probabilistic properties of the model to hold. Third, it is well-known in the literature that density based tests are more sensitive to local deviations of the model while distribution function based tests are more sensitive to global type of deviations to the null model; to account for different types of deviations to the null model, we consider both the density function based test and the distribution function based test, and study their finite sample power in Monte Carlo experiments.

An alternative way of formulating a test would be to base the test statistic on the conditional distribution of the observed returns. Although one might expect that conditional distribution based tests are superior to marginal distribution based tests as considered in this paper, there is no theoretical result nor empirical evidence to support this claim. Theoretically, we know that there is no uniformly most powerful test in nonparametric testing problems (see e.g. Aït-Sahalia et al., 2009 p. 1105). Empirically, as we argued earlier in this section, the marginal distribution of the observed returns by itself is an important object of empirical modelling and its specification would need to be checked usually in the first place. Therefore we view the tests based on the marginal distribution and the conditional distribution as complementary to each other — both types of tests contain information that will shed light on our

understanding of the source of (possible) misspecification of a model. Moreover, we also discuss the possible extension of our tests to

[2] That is, there might exist two different specifications $\{b(.;\theta), a(.;\theta)\}$ and $\{\tilde{b}(.;\theta), \tilde{a}(.;\theta)\}$ leading to the same return distribution with density $q(x)$.