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Testing against changing correlation

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ABSTRACT

A test for time-varying correlation is developed within the framework of a dynamic conditional score (DCS) model for both Gaussian and Student t-distributions. The test may be interpreted as a Lagrange multiplier test and modified to allow for the estimation of models for time-varying volatility in the individual series. Unlike standard moment-based tests, the score-based test statistic includes information on the level of correlation under the null hypothesis and local power arguments indicate the benefits of doing so. A simulation study shows that the performance of the score-based test is strong relative to existing tests across a range of data generating processes. An application to the Hong Kong and South Korean equity markets shows that the new test reveals changes in correlation that are not detected by the standard moment-based test.

1. Introduction

The possibility that the correlations between financial assets are changing over time is an important issue in many areas of finance, such as portfolio construction and risk management; see Lumsdaine (2009) for a recent discussion. The aim here is to provide a test for time-varying correlation that is powerful, yet simple to implement. The proposed approach is based on the dynamic conditional score (DCS) models recently developed by Creal et al. (2011, 2013) and Harvey (2013). It is shown that Lagrange multiplier (LM) tests can be constructed from the autocorrelations of the conditional scores, with a modified test taking account of estimated dynamic variances. Without this modification the test is based on a simple portmanteau statistic. The scores incorporate information on the level of correlation, and local power arguments indicate that the resulting test can be expected to be more powerful as the unconditional correlation moves away from zero. This is not the case with the standard moment-based portmanteau test, introduced by Bollerslev (1990), which simply uses the cross-product of standardized residuals.

The tests are developed for a bivariate Gaussian model, with a subsequent extension to the bivariate Student t-distribution. Monte Carlo experiments are used to compare the performance of these tests with existing tests, including those of Tse (2000, 2002) and Bera and Kim (2002). The results show that, on the whole, the proposed tests perform much better than existing tests across a range of data generating processes. Although the competing tests, which include portmanteau tests, residual regression tests and Lagrange multiplier tests, are based on a variety of approaches, they generally rely on the cross-product of standardized residuals to identify potential time variation and so share the same weakness relative to the scores. This point is highlighted by an application to the Hong Kong and

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South Korean equity markets, where it is found that the score-based tests can identify changing correlations that are undetectable by a moment-based test.

The paper is organized as follows. Section 2 reviews the bivariate DCS model for time-varying correlation and Section 3 shows how the new tests can be derived as LM tests within this framework. Section 4 presents the Monte Carlo results and Section 5 extends the theory and Monte Carlo study to the bivariate t-distribution. The application is reported in Section 6 and 7 concludes.

2. The DCS model for time-varying correlation

Consider a bivariate Gaussian model in which the observations, y_{1t} and y_{2t}, have zero means and constant variances, but the correlation between them changes over time. The covariance matrix is

$$\Sigma_{t|t-1} = \begin{bmatrix} \sigma_1^2 & \rho_{t|t-1}\sigma_1\sigma_2 \\ \rho_{t|t-1}\sigma_1\sigma_2 & \sigma_2^2 \end{bmatrix},$$

where $\rho_{t|t-1}$ denotes the correlation, which changes in a way that depends on information available at time $t-1$. Rather than working directly with $\rho_{t|t-1}$, a transformation is applied so as to keep it in the range, $-1 < \rho_{t|t-1} < 1$. The link function

$$\rho_{t|t-1} = \frac{\exp\left(2\gamma_{t|t-1}\right)-1}{\exp\left(2\gamma_{t|t-1}\right)+1}, \quad t = 1, \dots, T, \tag{1}$$

is eminently suitable in that it allows the new variable, $\gamma_{t|t-1}$, to be unconstrained.

The log-density of the t-th pair of observation, conditional on information at time $t-1$, is

$$\ln f(y_{1t}, y_{2t}; \psi, \lambda_1, \lambda_2) = -\ln 2\pi - \ln\sigma_1^2 - \ln\sigma_2^2 - \frac{1}{2}\ln\left(1-\rho_{t|t-1}^2\right) - \frac{1}{2\left(1-\rho_{t|t-1}^2\right)}\left(\frac{y_{1t}^2}{\sigma_1^2} - \frac{2\rho_{t|t-1}y_{1t}y_{2t}}{\sigma_1\sigma_2} + \frac{y_{2t}^2}{\sigma_2^2}\right),$$

where ψ denotes the parameters upon which $\rho_{t|t-1}$, and hence $\gamma_{t|t-1}$, depend. The score with respect to $\gamma_{t|t-1}$, that is $\partial \ln f_t/\partial\gamma_{t|t-1}$, can be written in terms of $\rho_{t|t-1}$ as

$$u_t = \frac{1}{4}(x_{1t}+x_{2t})^2\frac{1-\rho_{t|t-1}}{1+\rho_{t|t-1}} - \frac{1}{4}(x_{1t}-x_{2t})^2\frac{1+\rho_{t|t-1}}{1-\rho_{t|t-1}} + \rho_{t|t-1}, \tag{2}$$

where $x_{it} = y_{it}/\sigma_i$, $i = 1,2$. We can also write

$$u_t = \frac{1}{1-\rho_{t|t-1}^2}\left[\left(1+\rho_{t|t-1}^2\right)x_{1t}x_{2t} - \rho_{t|t-1}\left(x_{1t}^2 + x_{2t}^2\right)\right] + \rho_{t|t-1}. \tag{3}$$

It is not difficult to see that $E(u_t) = 0$.

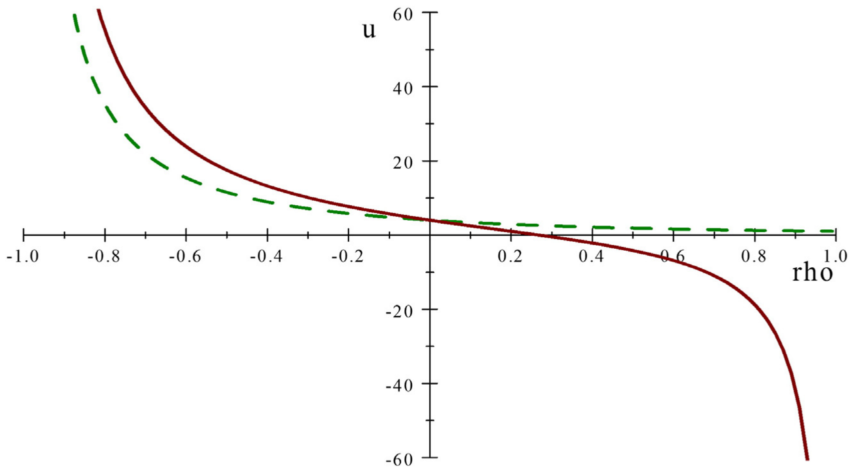

Fig. 1. Plot of score, u, against correlation, ρ, for $x_1 = x_2 = 2$ (dash) and $x_1 = 4$, $x_2 = 1$.