ARTICLE IN PRESS

EMPFIN-0885; No of Pages 26

Journal of Empirical Finance xxx (2016) xxx–xxx

Contents lists available at ScienceDirect

Journal of Empirical Finance

journal homepage: www.elsevier.com/locate/jempfin

Testing the martingale hypothesis for gross returns☆

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ARTICLE INFO

Article history:
Received 21 December 2014
Received in revised form 21 February 2016
Accepted 25 February 2016
Available online xxxx

JEL classification:
C10
C22
G10
G14

Keywords:
Variance ratio tests
Martingale
Predictability

ABSTRACT

We propose an alternative ratio statistic for measuring predictability of stock prices. Our statistic is based on actual returns rather than logarithmic returns and is therefore better suited to capturing price predictability. It captures not only linear dependence in the same way as the variance ratio statistics of Lo and MacKinlay (1988) but also some nonlinear dependencies. We derive the asymptotic distribution of the statistics under the null hypothesis that simple gross returns are unpredictable after a constant mean adjustment. This represents a test of the weak form of the Efficient Market Hypothesis. We also consider the multivariate extension, in particular, we derive the restrictions implied by the EMH on multiperiod portfolio gross returns. We apply our methodology to test the gross return predictability of various financial series.

1. Introduction

Variance ratio tests (Cochrane (1988); Lo and MacKinlay (1988); Poterba and Summers (1988)) are widely used to test the (weak form of) Efficient Market Hypothesis (EMH) of no predictability of asset returns. One particular advantage of the variance ratio test over the alternatives, such as the standard Box-Pierce statistic, is that the direction of the ratio depends on all the first K autocorrelations and their relative magnitudes, thus providing the direction of the predictability. The original variance ratio test, developed by Lo and MacKinlay (1988) and all other modifications thereof focus on the log return predictability, where the log return is defined to be the first difference of the log prices, i.e., $r_t := logP_t - logP_{t-1}$. Although very convenient, log returns are just an approximation of the actual return defined by $R_t := \frac{P_t}{P_{t-1}} - 1$, which is much harder to work with. Due to its convenience, most tests of the EMH were developed for the log returns. Here, we focus directly on the simple gross return $\mathcal{R}_t := \frac{P_t}{P_{t-1}}$ and derive alternative ratio statistics to test the hypothesis that risk adjusted gross returns are a martingale difference sequence. There are many discussions around which choice of return to use to measure performance and to fit asset pricing models. The main argument for logarithmic returns is mathematical simplicity and their continuously compounded interpretation that fits with continuous time models. From the point of view of the buy and hold investor though, the actual return over the relevant

☆ We thank Dario Bonciani, Steve Thiele and Mark Salmon for the helpful comments and suggestions and the Cambridge INET Institute for the financial support.
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http://dx.doi.org/10.1016/j.jempfin.2016.02.010
0927-5398/

Please cite this article as: O. Linton, E. Smetanina, Testing the martingale hypothesis for gross returns, Journal of Empirical Finance (2016), http://dx.doi.org/10.1016/j.jempfin.2016.02.010

horizon is what matters. In fact, standard economic theory requires that the appropriate measure of return used in deriving the cost of capital for one period, for example, should be $E(R_t)$, i.e., the true arithmetic mean. This requirement holds whatever the nature of the process that generates R_t. See, for example, the treatment in Copeland and Weston (1988) Chapters 7 and 13. The difference between these return measures can lead to substantial differences in practice for longer horizons, see for example Roll (1983) who argues that using buy and hold returns produces an estimated small firm premium only one half as large as that based on alternative methods. Mindin (2011) also investigates the difference between arithmetic and geometric returns. We exploit the implied scaling of gross returns that follows from a martingale assumption on prices to derive our test statistic. Under our null hypothesis and some mild additional conditions it satisfies a Central Limit Theorem, and we show how to conduct inference under the null hypothesis. In Section 2, we describe our null hypothesis and test statistic. In Section 3 we derive the limiting null distribution under two alternative sets of regularity conditions. In Section 4 we define asymptotic standard errors and and a bias correction based on asymptotic expansion. In Section 5 we define critical values based on subsampling method. In Section 6 we discuss two alternative hypotheses and how they influence the test statistic. In Section 7 we provide the theory for multivariate version of our test statistic. In Section 8 we present an application as well as size and power analysis of our univariate and multivariate test statistics. Section 9 concludes.

Throughout the paper "$\Rightarrow$" denotes convergence in distribution.

2. The null hypothesis and test statistic

Suppose that stock prices P_t obey the martingale hypothesis (after a constant risk adjustment which we take to be represented by μ), or more precisely suppose that the gross return series satisfies

$$E[\mathcal{R}_{t+1} | \mathcal{F}_t] = E\left[\frac{P_{t+1}}{P_t} | \mathcal{F}_t\right] = (1+\mu) \tag{1}$$

for each t, where $\mathcal{F}_t = \sigma(P_k, k \le t)$ is a sigma-algebra, containing current and past prices and μ is a constant. The gross return over the horizon t to $t+j$ can be written as

$$\mathcal{R}_{t+j}(j) = \frac{P_{t+j}}{P_t} = \frac{P_{t+j}}{P_{t+j-1}} \times \frac{P_{t+j-1}}{P_{t+j-2}} \times \cdots \times \frac{P_{t+1}}{P_t} = \mathcal{R}_{t+1} \times \mathcal{R}_{t+2} \times \ldots \times \mathcal{R}_{t+j}, \tag{2}$$

which is also the buy and hold return for horizon j, Roll (1983). By the law of iterated expectations it follows that

$$E[\mathcal{R}_{t+j}(j) | \mathcal{F}_t] = (1+\mu)^j \equiv \mu_j \tag{3}$$

for all $j \in \mathbb{Z}$ and all t. Hence, the unconditional means satisfy $E[\mathcal{R}_{t+1}] = (1+\mu)$ and $E[\mathcal{R}_{t+j}(j)] = (1+\mu)^j$. We consider the following ratio

$$\tau_K = \frac{E[\mathcal{R}_{t+K}(K)]}{E^K[\mathcal{R}_{t+1}]} = 1. \tag{4}$$

This ratio is the basis of our testing strategy. Unlike the usual variance ratio statistics, this quantity only depends on the first moments of gross returns, but we show below how this quantity captures linear dependence under the alternative hypothesis. In fact there is a more general class of statistics $\tau_{K,L,\alpha,\beta}$, which can be written as

$$\tau_{K,L,\alpha,\beta} \equiv \frac{(E[\mathcal{R}_{t+K}(K)])^{\alpha}}{(E[\mathcal{R}_{t+L}(L)])^{\beta}} = 1, \tag{5}$$

where $\beta/\alpha = K/L$. We mostly focus on τ_K and $\tau_{K,1,1/K,1}$. We next turn to estimation.

Suppose that we observe a sample of prices on an unequally spaced grid $\{t_1, \ldots, t_T\}$, P_{t_i}, $i = 1, \ldots, T$. Define the spacing of the observations $\delta_i = t_{i+1} - t_i \in \mathbb{Z}_+$, for $i = 2, \ldots, T$; regular sampling would have $\delta_i = 1$ for all i, but other structures are encountered in practice. Then define for $j = 1, 2, \ldots$

$$\hat{\mu}_j = \frac{1}{T_j} \sum_{\{i:\delta_i = j\}} \frac{P_{t_{(i+1)}}}{P_{t_i}} = \frac{1}{T_j} \sum_{\{i:\delta_i = j\}} \frac{P_{t_i+j}}{P_{t_i}}, \tag{6}$$

where $T_j = \sum_{i=1}^{T-1} 1\{\delta_i = j\}$ is the number of observations available to compute the j period return. In the special case that the observations are equally spaced, the spacing is $\delta_i = t_{i+1} - t_i = 1$. Then define for $j = 1, 2, \ldots)$

$$\hat{\mu}_j = \frac{1}{T-j} \sum_{t=1}^{T-j} \frac{P_{t+j}}{P_t}.$$

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