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Analysis and benchmarking of meta-heuristic techniques for lay-up optimization

Mark W. Bloomfield *, J. Enrique Herencia, Paul M. Weaver

Department of Aerospace Engineering, University of Bristol, Bristol, BS8 1TR, UK

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ABSTRACT

The analysis and benchmarking of three meta-heuristic algorithms to determine laminate stacking sequences is presented. The benchmarking is undertaken for a simply supported composite laminate subject to strength and buckling constraints. A genetic algorithm, ant colony and particle swarm optimization are considered. It is shown that for inherently discrete sets of ply orientations, ant colony optimization outperforms the other algorithms. For continuous problems, a particle swarm may be the most appropriate. It is further shown that selection of an appropriate algorithm may be indeed problem dependent.

1. Introduction

Driven by environmental and economic targets, there is a greater need for low weight structures in civil and military aircraft. As such, the aviation industry is rapidly employing composite materials for primary structures such as wings and fuselages. The excellent performance of composite materials has been well publicised in recent years. Studies have shown they possess excellent stiffness and strength properties [1,2]. Despite their insertion in high profile aircraft (e.g. Boeing 787), there are potential efficiency gains to be made by undertaking stacking sequence (lay-up) optimization and reducing dependence on the use of homogeneous properties, or "black metal".

Lay-up optimization of laminated composites has evolved significantly over the past 25 years. For further details, see Ref. [3]. Ghiasi et al. [4] recently provided a detailed review of the various optimization techniques which have been successfully applied to laminated composite design optimization. Recent focus has been on the optimization of laminated composites using lamination parameters and/or meta-heuristic approaches. A two level optimization approach has recently been adopted by Herencia et al. [3,5] and Bloomfield et al. [6,7] to solve the stacking sequence problem. At the first level, gradient based methods were used to determine optimal lamination parameters and plate thicknesses. All constraints such as strength and buckling were embedded at this level and necessary trade-offs considered. Lamination parameters are particularly useful intermediate design variables in the optimization of laminated composites because the constraining relationships between lamination parameters form a convex feasible region [8,9]. Consequently, where the objective function and constraints are a convex function (in minimization problems) of the design variables, gradient based methods guarantee that global optima are obtained. At the second level of the optimization a meta-heuristic optimizer was used to obtain a stacking sequence which satisfies the set of design constraints. It is important to note that at the second level the fitness function is highly non-convex. The non-convexity of the fitness function arises due to the mapping between ply orientations, lamination parameters and the fitness function. As such, gradient based methods may only find local optima and thus not entirely appropriate. Motivated by this shortfall several meta-heuristic optimization algorithms are considered. Specifically, a genetic algorithm (GA), ant colony optimization (ACO) and particle swarm optimization (PSO) are used. The chief benefit of using a meta-heuristic algorithm is that gradient information is not required. Whilst local optima remain a problem, the ability to escape local optima can be studied and certain parameters adjusted to improve the performance of each method.

Lay-up optimization is inherently discrete. In laminate design, ply thickness is generally fixed and ply orientations take a range of discrete values. Determining a stacking sequence of a given plate thickness using ply orientations as design values is a combinatorial problem. For a laminate of n plies where each ply orientation can take a value in a set of size m, it follows that the number of possible designs is m^n. As such, enumeration quickly becomes increasingly difficult due to the combinatorial explosion of possible lay-ups. To overcome this problem, population based meta-heuristics have again been proposed. The success of GAs in composite optimization

* Corresponding author.
E-mail address: aemwb@bristol.ac.uk (M.W. Bloomfield).

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Nomenclature

A_{ij}	components of the in-plane stiffness matrix
A	cross sectional area of the plate
a	plate length
b	plate width
c_i	weighting factors
D_{ij}	components of the out-of-plane stiffness matrix
E_{11}, E_{22}	longitudinal and transverse Young's moduli
F	fitness function
G_{12}	shear modulus
h	plate thickness
M	vector of out-of-plane moments
n_i	number of each ply orientation in the lay-up
N	vector of in-plane loads
N_i	load per unit width in the i direction
Q_{ij}	components of the reduced stiffness matrix
r_i	random weighting factors
U_i	material invariants
s	symmetric
$v_{ij}(t)$	velocity of particle i in dimension j at discrete time step t
$x_i(t)$	position of particle i at discrete time step t
$y_i(t)$	local best position of particle i discrete time step t
$\hat{y}(t)$	global best position at discrete time step t
z	normalized through thickness co-ordinate
α_{ij}	ant routing table
ε^0	vector of mid-plane strains
$\xi_i^{A,D}$	lamination parameters ($i = 1 \ldots 4$)
κ	vector of plate curvatures
τ_{ij}^k	pheromone deposited by ant k on arc ij
ρ	density of material
$\widehat{\rho}$	pheromone evaporation coefficient
ν_{12}	Poisson's ratio
φ_i	ith design envelope of ply orientations
ω	swarm inertia coefficient

have been well documented. For example, Le Riche and Haftka [10], Nagendra et al. [11], Liu et al. [12], and Herencia et al. [13,14] have all successfully applied GAs, both directly and as a multi-level approach, to composite optimization. Moreover, GAs naturally lend themselves to discrete variables as each gene in a GA can represent a single angle in a lay-up. Despite the popularity of GAs, they may be computationally less efficient than other methods and often require extensive parameter refinement to ensure timely convergence. In contrast, particle swarm optimization (PSO), inspired by the notion of birds flocking (Kennedy and Eberhart [15]) may require less computational effort and parameter refinement. Initially designed for continuous domains, the paradigm has been successfully applied to discrete problems such as lay-up optimization. Bloomfield et al. [7] demonstrated the suitability of the PSO with respect to lay-up optimization using discrete variables by rounding the continuous values to the nearest discrete point in the set of possible ply orientations. Furthermore, Bloomfield et al. [7] reduced the number of parameters of the standard PSO model by introducing a new random parameter. The authors showed efficiency gains over the standard PSO model by encouraging and maintaining diversity in the swarm. Suresh et al. [16] further highlighted the benefits of a discrete PSO applied to a multi-objective composite box design and highlighted the gains in using a PSO over a GA. Kathiravan and Ganguli [17] recently used a gradient based method and PSO to determine laminate stacking sequences to satisfy strength failure criteria in a box beam. The analysis highlighted that a PSO was able to determine a global optima, whereas a gradient based optimizer would only converge to local optima. Despite this shortcoming, the local optima were often good points and could be used as starting points in the PSO. Omkar et al. [18] successfully applied a variant of the standard PSO vector evaluated particle swarm optimization (VEPSO) model introduced by Kennedy and Eberhart [15]. The authors solved a multiple objective problem of minimizing weight and the total cost of the composite structure subject to three failure criteria: failure mechanism based failure criteria, maximum stress failure criteria and the Tsai–Wu failure criteria. Recently, the ant colony optimization (ACO) paradigm [19] has been receiving growing interest. It is noted that the ACO paradigm is specifically designed for combinatorial problems such as stacking sequence optimization. This is in direct contrast to the PSO which was designed for continuous domains. Aymerich and Serra [20,21] demonstrated the effectiveness of the ACO in maximizing the buckling load of a composite plate (with different boundary conditions) subject to strength constraints. The authors also showed significant efficiency gains over a GA. Bloomfield et al. [7] also used an ACO to determine laminate stacking sequences subject to strength and buckling constraints. In Ref. [21], it was shown that a colony could rapidly determine stacking sequences and, furthermore, that this approach was robust and reliable. Naturally, the implementation of the aforementioned optimization algorithms can dramatically affect the performance of the algorithm. Whilst a number of variants of each meta-heuristic approach exist, the versions which are compared and contrasted are those given in Ref. [13,21,7] for the GA, ACO and PSO, respectively. These are chosen as they represent current state-of-the-art implementations of the aforementioned techniques.

In the paper, the GA, PSO and ACO are analyzed and compared in detail. The analysis aims to show two key points, (a) how the number of possible ply orientations affects convergence and (b) how does laminate thickness (complexity due to number of plies) affect convergence. Through an efficiency, reliability and robustness analysis, it will be shown that an ACO and PSO offer the best routes to determining laminate stacking sequences. It is further observed that the selection of the optimization technique may be problem dependent.

2. Laminate constitutive equations

Tsai et al. [1] and Tsai and Hahn [2] characterized the stiffness properties of laminated composites as linear functions of material invariants and at most 12 lamination parameters. Since the plate is symmetric and thus does not exhibit any bending-extension coupling, the constitutive equation for a symmetric laminated plate using classical laminate theory (e.g. Ref. [2]) is given, by,

$$\begin{Bmatrix} N \\ M \end{Bmatrix} = \begin{bmatrix} A & 0 \\ 0 & D \end{bmatrix} \begin{Bmatrix} \varepsilon^0 \\ \kappa \end{Bmatrix} \tag{1}$$

where N is the vector of in-plane loads, M is a vector of resultant out-of-plane moments, ε^0 is the vector of mid-plane strains and κ is the vector of plate curvatures. The in-plane and out-of-plane stiffness matrices are defined in terms of lamination parameters and material invariants,