Computers and Structures 88 (2010) 322–330

Contents lists available at ScienceDirect

Computers and Structures

journal homepage: www.elsevier.com/locate/compstruc

Dynamic torsional buckling of cylindrical shells

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ARTICLE INFO

Article history:
Received 5 June 2009
Accepted 16 November 2009
Available online 22 December 2009

Keywords:
Cylindrical shell
Dynamic buckling
Hamiltonian system
Local buckling
Torsion

ABSTRACT

In this paper the local buckling of cylindrical shell under torsion is discussed. The Hamiltonian system approach is employed to analyze the propagation of shear wave. In this system, critical torsional loads and buckling modes are reduced to a problem of eigenvalues and eigensolutions of increasing orders. Buckling modes are divided into two types, the local torsional buckling modes and the helical buckling modes. For short-time shear wave propagation, local torsional buckling occurs easily. On the contrary, the helical buckling modes appear when the wave propagates for a longer time and these modes correspond to the first-order eigensolution.

1. Introduction

Cylindrical shells are simple yet very important structural components in engineering applications. The stability of shell structures, e.g. shell buckling, is a fundamental problem in engineering design. Many papers have described the deformation of different kinds of cylindrical shells under various loadings. For example, Ross et al. [1] and Blachut and Jaiswal [2] investigated cylinders under external pressure through numerical and experimental approaches and theoretical methods. Cylindrical shells under the action of uniform axial compression and air blast loading conditions were presented [3,4].

In this context, the behavior of torsional buckling is an interesting issue but it receives relatively little attention than compression buckling. Pioneering works on approximate solutions for shell buckling were reported by Donnell [5]. Lundquist [6] and Nash [7] did some buckling experiments but it was Yamaki [8] who conducted precise experimental studies and reported approximate solutions that were in reasonable agreement with experimental results. These prior works were restricted to elastic static buckling of cylindrical shells. In recent years, there were more studies on the dynamic torsional buckling of cylindrical shells. There were some analyses for torsional buckling of cylindrical shells or bars made of different materials [9–13] and Mao and Lu [15] studied the elastic–plastic buckling with a deep thick-shell model. Buckling and post-buckling of cylindrical shell was also considered [16–18]. Experimental studies were carried out by Ma et al. [19] on dynamic plastic buckling of circular cylindrical shells under impact torque. The experimental results of static plastic torsional buckling of circular cylindrical shells were presented and discussed. Cylindrical shells subjected to combined axial and torsion loading were also investigated by a numerical approach and also in an experiment [20–23].

One of the buckling modes for long cylindrical shells is the purely flexural mode similar to the classical buckling of a straight rod. Such flexural buckling is well understood. Other aspects such as local buckling and global buckling of cylindrical shells under axial impact are discussed in many papers. Silvestre [24] discussed the buckling behavior of elliptical cylindrical shells and tubes under compression. He analysed the local and global buckling behavior of elliptical hollow section (EHS) members which were subjected to compression in an effort to illustrate the formulation of Generalised Beam Theory (GBT). By quasi-static and dynamic tests, Jensen et al. [25] studied the transition between progressive and global buckling of axially loaded square aluminium tubes. Energy absorption was found to be dependent very much on the collapse mode. Paimushin [26] reported details of local and global buckling under combined loads. He showed the existence of previously unknown torsional, flexural, and torsional–flexural buckling modes for cylindrical shells which were subjected to simultaneous compression and external pressure. He also concluded that before the classical buckling took place, the torsional buckling for relatively short shells with low shear rigidity in the tangent plane could occur. Other buckling modes occured for relatively long shells. Comparatively, research for local and global torsional buckling for cylindrical shells has been very insufficient.

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doi:10.1016/j.compstruc.2009.11.005

Nomenclature

c	$c \approx \sqrt{E/[2\rho(1+\mu)]}$, wave speed	$\mathbf{q}$, $\mathbf{p}$	mutually dual vectors
D, K	$K = Eh/(1-\mu^2)$, $D = Eh^3/12(1-\mu^2)$, shell flexural rigidity and stiffness	(r,θ,x)	circular cylindrical coordinate
E	Young's modulus	$T_{x\theta}$	external moment or torsion
H	Hamiltonian function	(w,v,u)	displacements
$\mathbf{H}$	Hamiltonian operator matrix	ε_x, ε_θ, $\varepsilon_{x\theta}$	membrane strains and shear strains on the neutral surface
k_x, k_θ, $k_{x\theta}$	bending and torsional strains on the neutral surface	μ	Poisson's ratio
$\tilde{L}$	Lagrange function	ρ	material density
M_x, M_θ, $M_{x\theta}$	bending and torsional moments		
N_x, N_θ, $N_{x\theta}$	internal stresses and shear stresses		

Unlike the traditional solution methodology using the Lagrange system formulation in the Euclidian space, Xu et al. [27–29] developed a new Hamiltonian system approach to study buckling of shell under axial impact [29–32]. New symplectic elasticity approach has also been applied for deriving exact analytical solutions of beams and rectangular plates [33–36]. In this paper, the Hamiltonian system approach is generalized to study dynamic buckling of cylindrical shell under torsional impact. The factors which influence the buckling modes are fully investigated and discussed.

2. Hamiltonian system and dual equations

Consider an homogeneous elastic cylindrical shell (Fig. 1) with radius r, thickness h, length l, Young's modulus E and Poisson's ratio μ.

Using the circular cylindrical coordinate system (r,θ,x), the constitutive relations can be expressed as

$$\begin{cases} N_x = K\left[\frac{\partial u}{\partial x} + \frac{\mu}{r}\left(\frac{\partial v}{\partial\theta} + w\right)\right] \\ N_\theta = K\left[\frac{1}{r}\left(\frac{\partial v}{\partial\theta} + w\right) + \mu\frac{\partial u}{\partial x}\right], \\ N_{x\theta} = \frac{K(1-\mu)}{2}\left(\frac{1}{r}\frac{\partial u}{\partial\theta} + \frac{\partial v}{\partial x}\right) \end{cases} \quad \begin{cases} M_x = -D\left[\frac{\partial^2 w}{\partial x^2} + \frac{\mu}{r^2}\left(\frac{\partial^2 w}{\partial\theta^2} - \frac{\partial v}{\partial\theta}\right)\right] \\ M_\theta = -D\left[\frac{1}{r^2}\left(\frac{\partial^2 w}{\partial\theta^2} - \frac{\partial v}{\partial\theta}\right) + \mu\frac{\partial^2 w}{\partial x^2}\right], \\ M_{x\theta} = -\frac{D(1-\mu)}{r}\left(\frac{\partial^2 w}{\partial x\partial\theta} - \frac{1}{2}\frac{\partial v}{\partial x}\right) \end{cases} \tag{1}$$

where $D = Eh^3/[12(1-\mu^2)]$ and $K = Eh/(1-\mu^2)$. The Lagrange function, which consists of membrane energy, bending energy, kinetic energy and work of external load, is

$$\begin{aligned}\tilde{L} = {} & \frac{K}{2}\left[\left(\frac{\partial u}{\partial x}\right)^2 + \frac{1}{r^2}\left(\frac{\partial v}{\partial\theta} + w\right)^2 + \frac{2\mu}{r}\frac{\partial u}{\partial x}\left(\frac{\partial v}{\partial\theta} + w\right) + \frac{1-\mu}{2}\left(\frac{1}{r}\frac{\partial u}{\partial\theta} + \frac{\partial v}{\partial x}\right)^2\right] \\ & + \frac{D}{2}\left[\left(\frac{\partial^2 w}{\partial x^2}\right)^2 + \frac{1}{r^4}\left(\frac{\partial^2 w}{\partial\theta^2} - \frac{\partial v}{\partial\theta}\right)^2 + \frac{2\mu}{r^2}\frac{\partial^2 w}{\partial x^2}\left(\frac{\partial^2 w}{\partial\theta^2} - \frac{\partial v}{\partial\theta}\right) + \frac{2(1-\mu)}{r^2}\left(\frac{\partial^2 w}{\partial x\partial\theta} - \frac{1}{2}\frac{\partial v}{\partial x}\right)^2\right] \\ & + \frac{T_{x\theta}}{r}\frac{\partial w}{\partial x}\frac{\partial w}{\partial\theta} - \frac{1}{2}\rho h\left(\frac{\partial u}{\partial t}\right)^2 - \frac{1}{2}\rho h\left(\frac{\partial v}{\partial t}\right)^2 - \frac{1}{2}\rho h\left(\frac{\partial w}{\partial t}\right)^2,\end{aligned} \tag{2}$$

where $T_{x\theta}$ is the external moment or torsion. Bases on the Hamiltonian principle, the equation of variation is expressed as

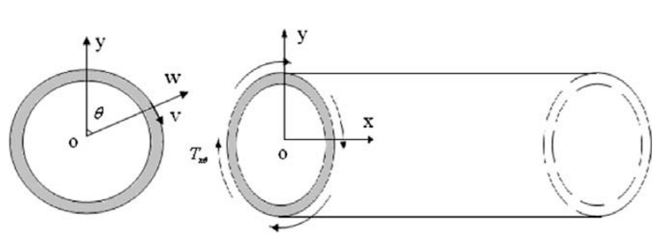

Fig. 1. Geometrical parameters of a cylindrical shell with torsional impact.

$$\delta\int_{t_0}^{t_1} dt \int_0^{2\pi} d\theta \int_0^l L(u,v,w)dx = 0. \tag{3}$$

From Eq. (3), the governing equations in the Lagrangian system can be obtained directly as [14,15]

$$\begin{cases} K\left[\frac{\partial^2 u}{\partial x^2} + \frac{\mu}{r}\left(\frac{\partial^2 v}{\partial x\partial\theta} + \frac{\partial w}{\partial x}\right) + \frac{1-\mu}{2}\left(\frac{1}{r^2}\frac{\partial^2 u}{\partial\theta^2} + \frac{1}{r}\frac{\partial^2 v}{\partial x\partial\theta}\right)\right] = \rho h\frac{\partial^2 u}{\partial t^2}, \\ K\left[\frac{1}{r^2}\left(\frac{\partial^2 v}{\partial\theta^2} + \frac{\partial w}{\partial\theta}\right) + \frac{\mu}{r}\frac{\partial^2 u}{\partial x\partial\theta} + \frac{1-\mu}{2}\left(\frac{1}{r}\frac{\partial^2 u}{\partial x\partial\theta} + \frac{\partial^2 v}{\partial x^2}\right)\right] \\ \quad - \frac{D}{r^2}\left[\frac{1}{r^2}\left(\frac{\partial^3 w}{\partial\theta^3} - \frac{\partial^2 v}{\partial\theta^2}\right) + \mu\frac{\partial^3 w}{\partial x^2\partial\theta} + (1-\mu)\left(\frac{\partial^3 w}{\partial x^2\partial\theta} - \frac{1}{2}\frac{\partial^2 v}{\partial x^2}\right)\right] = \rho h\frac{\partial^2 v}{\partial t^2}, \\ -K\left[\frac{1}{r^2}\left(\frac{\partial v}{\partial\theta} + w\right) + \frac{\mu}{r}\frac{\partial u}{\partial x}\right] - D\left[\frac{\partial^4 w}{\partial x^4} + \frac{2}{r^2}\frac{\partial^4 w}{\partial x^2\partial\theta^2}\right. \\ \quad \left. + \frac{1}{r^4}\left(\frac{\partial^4 w}{\partial\theta^4} - \frac{\partial^3 v}{\partial\theta^3}\right) - \frac{1}{r^2}\frac{\partial^3 v}{\partial x^2\partial\theta}\right] + \frac{2T_{x\theta}}{r}\frac{\partial^2 w}{\partial x\partial\theta} = \rho h\frac{\partial^2 w}{\partial t^2}. \end{cases} \tag{4}$$

As a result of cylindrical shells under impact torque, displacements u and w can be ignored prior to shell buckling as compared to displacement v in the second equation of Eq. (4). For uniform torsional impact at the end of shell, the torsional wave propagates along the central axis before buckling occurs. Hence the torsional wave equation is

$$c^2\frac{\partial^2 v}{\partial x^2} - \frac{\partial^2 v}{\partial t^2} = 0, \tag{5}$$

where $c = \sqrt{E/[2\rho(1+\mu)]}\sqrt{1+(h/r)^2/12}$ is the wave velocity. For a thin shell, $c \approx \sqrt{E/[2\rho(1+\mu)]}$. The solution of Eq. (5) can be obtained easily and the distribution of shear stress can be derived at the same time.

The following dimensionless parameters $X = x/r$, $W = w/r$, $U = u/r$, $V = v/r$, $L = l/r$, $T_{cr} = T_{x\theta}r^2/D$ and parameters $\gamma = 12(r/h)^2$, $T = ct/r$ are adopted. Let an over-dot denote differentiation with respect to θ, namely $\dot{(\ )} \equiv \partial(\ )/r\partial\theta$ in which the θ-coordinate can be taken as a time-equivalent coordinate, $\partial(\ )/\partial X \equiv \partial_X(\ )$ and $\partial(\ )/\partial T \equiv \partial_T(\ )$. Because shell buckling with perturbation equations is considered, the kinetic energy can be ignored. Let the bending angle be $\varphi = -\dot{W} + V$, such that the dimensionless Lagrange function can be expressed as

$$\begin{aligned}\tilde{L} = {} & \frac{\gamma}{2}\left\{(\partial_X U + \dot{V} + W)^2 - 2(1-\mu)\left[\partial_X U(\dot{V} + W) - \frac{1}{4}(\dot{U} + \partial_X V)^2\right]\right\} \\ & + \frac{1}{2}\left\{(\partial_X^2 W - \dot{\varphi})^2 + 2(1-\mu)\left[\dot{\varphi}\partial_X^2 W + \left(\partial_X\varphi - \frac{1}{2}\partial_X V\right)^2\right]\right\} \\ & + (\partial_X^2\dot{W} - \ddot{\varphi})(-\dot{W} + V - \varphi) + T_{cr}\dot{W}\partial_X W - \frac{1}{4}(1+\gamma)(1 \\ & - \mu)(\partial_T V)^2.\end{aligned} \tag{6}$$