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Formulation of an efficient hybrid time–frequency domain solution procedure for linear structural dynamic problems

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ABSTRACT

This work presents the detailed formulation of a hybrid time–frequency domain Green approach method for the solution of structural dynamic problems. A step-by-step time-domain solution procedure is established, based on the convolution between the Green functions of the problem and the vector of external loads. The Green functions are implicitly calculated in the frequency domain. The accuracy is significantly improved when compared with traditional direct integration methods, or with other methods based on the Green approach. This is due to the accurate calculation of the Green functions in the frequency domain, and to enhancements in the representation of the convolution integral.

1. Introduction

1.1. Context: time-domain solution procedures

Classical numerical integration methods for the solution of equilibrium equations in dynamic analysis are based on the use of finite-difference operators (or arbitrarily defined functions), that define the variation of the unknown values of displacements, velocities and accelerations within each time interval [1]. For instance, methods based on the Newmark family [1–5] have been successfully employed to solve linear and nonlinear problems, with physical or geometric properties that may be constant or vary with time and/or with the deformation of the structure.

Recent developments in time integration methods encompass several distinct lines of work, including for instance energy-conserving algorithms that present better stability behavior in severe nonlinear problems [6–9]; and also dissipative schemes to address the issues associated with higher frequencies [10–16]. Since the literature in these areas is quite vast, it would not be possible to cite all related works published over the years; therefore the reader is encouraged to look further into the references provided here, including also [17–19] where detailed overviews of past historical developments may be found.

A good illustration of practical applications of time integration algorithms is provided by the Brazilian experience in the offshore oil production industry. The assessment of very large deepwater oil fields in the Campos Basin has been motivating studies on new alternatives for structural systems to support deepwater platforms, leading to the development of fixed compliant structures [20], and of moored floating platforms [21], whose behavior is characterized by large displacements and therefore presents severe nonlinear dynamic effects. These are typical examples of the so-called "inertial" or "structural dynamic" problems, where the response is dominated by the mode shapes with lower frequency values. In general, the dynamic response of inertial problems is more efficiently attained with implicit time integration algorithms [1,5,22]. In this context, the authors have been directing research efforts to the development of time-domain computational strategies that present improved efficiency for the nonlinear dynamic analysis of structures, that are inherently time-consuming (see for instance [23–26]).

However, many systems considered in offshore applications present not only severe nonlinear effects, but also frequency-dependent mass, damping and stiffness properties. This characteristic is associated not only to the intrinsic structural behavior of the system, but also to the hydrodynamic effects due mainly to the fluid–structure interaction at the platform hull that must be considered in the calculation of environmental wave loadings. Therefore, for such applications the ability of the solution method to

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deal with frequency-dependent properties would be an added premium: current standard time-domain methods do not possess this ability.

1.2. Frequency-domain solution procedures

It is well known that, for linear problems, the dynamic equilibrium equations can be efficiently solved using domain transformation procedures (e.g. Laplace, Wavelet and Fourier transforms). The main advantage of such procedures, comparing with the standard time-domain approach, is precisely the low computational costs and the ability to deal with frequency-dependent properties.

For instance, frequency-domain methods based on the Discrete Fourier Transform (DFT) and Fast Fourier Transform (FFT) [27,28] are powerful tools for solving structural dynamic problems, particularly useful for applications to problems with frequency-dependent properties, and/or when spectral responses are desired.

However, such methods present several limitations. For instance, they cannot solve undamped problems; they are not particularly suited to deal with nonlinear problems, or even with linear transient problems in which the fundamental period of the Discrete Fourier Transform is not sufficiently extended [27].

1.3. Hybrid solution procedures

Following this line of research, solution procedures that combine time and frequency domain methods have been proposed, trying to combine computational efficiency and the ability of dealing with frequency-dependent properties and time-dependent nonlinear effects [29–32].

In 1991–92 a breakthrough was reached when Venancio-Filho and Claret [33,34] presented a matrix formulation for the analysis of SDOF systems in the frequency domain. This formulation served as the basis for the development of the ImFT (Implicit Fourier Transform) method, which was later generalized as a "time-segmented" or step-by-step procedure to consider MDOF systems, and also associated to a reduction method to solve dynamic problems in modal coordinates [35–37].

It can be demonstrated [37] that the ImFT approach corresponds to the standard convolution procedure of the Duhamel integral [27], where the unit-impulse response function or *Green function* can implicitly incorporate the frequency-dependent properties. In this approach, which is based on the Discrete Fourier Transform (DFT), the implicit calculation of the Green function is performed in the frequency domain. Later, this approach was extended to MDOF systems with nodal coordinates (not requiring reduction techniques), comprising the so-called ImFGA method (Implicit Frequency-Domain Green Approach) presented in [38], based on the calculation of Green function matrices.

Such methods are able to solve dynamic problems implicitly in the frequency domain, without explicitly calculating response spectra or transforming the domain of the external loading; the response is presented directly in the time domain. This way, frequency-dependent properties could be implicitly treated in the frequency domain. However, while mild nonlinear effects due to internal or external time-varying forces can be treated as pseudo-forces on the right-hand side of the equations of motion, these methods are not suited for strongly nonlinear problems.

Other approaches, related to the use of "step-response matrices" following the line introduced in [39] (see for instance [40]), can also be viewed as similar to these "Green function matrices" approach. Both approaches may be seen as derivations of the "piecewise exact methods" described in [27]. However there are significant differences between these methods, mainly related to the computation procedure and application of such matrices, as will be commented later in this work.

Extensions and variations of Green function methods have been recently proposed [41,42], based on *explicit* computation of the Green functions in the time-domain by a time integration algorithm. Those methods were shown to be efficient for medium-sized linear problems, retaining unconditional stability while presenting positive characteristics of explicit methods. However, as will be commented later in this work, such methods do not deal with frequency-dependent properties (noting however that they were not originally concerned with such class of problems).

1.4. Objective of the paper

In this context, the objective of this work is to present the detailed formulation of an efficient hybrid time–frequency domain solution procedure that is an evolution of previous methods based on Green functions [43]. This procedure will be referred here as the HTF-GA method (Hybrid Time–Frequency method with Green Approach).

The following sections of the paper begin by briefly commenting basic aspects of the formulation of the equations of motion for dynamic structural problems, including the basic scheme of direct time-domain integration methods and of the hybrid time–frequency domain integration method described here.

Next, the conceptual definition of the Green function is presented for SDOF problems, and then generalized for MDOF problems, leading to a recursive step-by-step solution procedure in the *time domain,* based on the convolution of the Green function matrices with the vector of external loading, added to initial condition terms also affected by the Green function matrices. Section 5 presents improved techniques for the calculation of the convolution integral, which are included amongst the new developments incorporated in the formulation described here.

Section 6 presents a detailed description of the procedures employed to compute the Green functions, which will be *implicitly* obtained in the *frequency domain* in terms of its steady-state harmonic components defined by a Fourier transform. Important aspects related to causality and convergence of the Fourier series are addressed.

Section 7 begins by describing a computational implementation of the HTF-GA method specialized for linear systems, and commenting on the computational costs involved. Next, extensions of the method for systems with nonlinear effects that can be treated with a pseudo-force technique are discussed. This section concludes with studies on the algorithmical properties of stability and accuracy.

Finally, results of the application of the method to simple problems are presented, in order to illustrate the potential of the method to obtain results with improved accuracy and computational efficiency.

2. Formulation of the equations of motion

It is well known [1,5] that the spatial discretization via the Finite Element Method yields a set of second-order "semi-discrete" ODE–Ordinary Differential Equations, which in turn can be discretized and solved in time by an appropriate integration algorithm. For linear problems, this ODE corresponds to the equations of motion:

$$\mathbf{M}\ddot{\mathbf{u}}(t) + \mathbf{C}\dot{\mathbf{u}}(t) + \mathbf{K}\mathbf{u}(t) = \mathbf{F}(t) \tag{1}$$

where $\ddot{\mathbf{u}}(t)$, $\dot{\mathbf{u}}(t)$ and $\mathbf{u}(t)$ are vectors containing the unknown nodal values of, respectively, accelerations, velocities and displacements; $\mathbf{F}(t)$ is the vector of external loads; $\mathbf{M}$ and $\mathbf{K}$ are the mass and stiffness matrices. Damping effects may be introduced through a Rayleigh proportional damping matrix $\mathbf{C}$ [1].