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Adaptive finite element based shape optimization in laminated composite plates

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ABSTRACT

In this study a reliable shape optimization for laminated plate structures has been attempted. For a fixed higher order plate model, a simple a-posteriori strain recovery algorithm, following ZZ type patch recovery technique, has been developed. The recovery is seen to be accurate. The effect of higher approximation order and mesh refinement on the quality of the obtained solution quantities like stress components and displacements, is studied in detail. The shape of the cutout is optimized with weight minimization as the objective function and the first-ply failure criterion as the constraint. It is observed that control of the discretization error (via adaptive mesh refinements) leads to vastly different final designs, as compared to those obtained using reasonably refined meshes, but without adaptivity. It is seen that without adaptivity, the design obtained is unsafe, as either more material removal is predicted or failure is predicted at higher loads, as compared to that obtained using adaptivity.

1. Introduction

Composite materials are finding wide applications in critical structural applications due to their capability of giving desired enhanced properties. Moreover, one can tailor these properties according to the requirements. These materials have very high strength to weight ratios. Cutouts in these critical structural components are inevitable. For example, in aerospace applications cutouts are made in wing ribs to facilitate the easy passage of fuel. Sometimes the cutouts are made to provide access for damage inspection or electrical circuits. In aerospace applications, weight saving is one of the important design criteria. Therefore, the cutouts are made just to reduce the weight of the structures. Since, these components are used for critical applications one should have confidence in the design procedure adopted.

A typical optimization based designing procedure involves evaluation of an objective function subjected to one or more constraints. For example, the objective function could be cost or weight minimization or profit maximization. Optimization problems, from engineering discipline, involve evaluation of the constraints which may include state of stress at a point; or a function which is a combination of stress components; deflection at a particular point; thermal stresses; buckling load, etc. Accurate computation of these constraints plays an important role in a reliable optimum design. In engineering design and optimization of large sized critical components, the use of finite element technique has become an integral part. Therefore, accurate evaluation of the finite element data, which in turn, is used in constraint evaluation, is very important.

Many researchers have attempted to optimize the composite laminate with design variables like ply thickness and ply orientation in order to obtain minimum weight designs subjected to several constraints, such as maximum deflection, maximum strength, maximum stress, von-Mises stress, first ply failure load (or reliability requirements), etc. (see [1–4]). The optimum design of laminated plates for maximum buckling load has also been attempted in [5–8] with constraint on the natural frequency. Botkin [9] has worked on shape optimization of stamped sheet metal parts with buckling and stress constraints. Sometimes the cutouts are just unavoidable in laminated structures. Hence, the shape optimization of laminated plates with cutouts for weight minimization has gained importance. For example, one can see the work on the optimization of composite plates with a cutout in [10,11]. Sivakumar et al. [12] have worked on optimization with dynamic constraints.

A survey on structural optimization can be seen in [13,14]. The optimization of aerospace structures with minimum weight objective, subjected to various constraints is reviewed in [15,16].

In general, the focus in all the studies mentioned above has been to demonstrate the effect of optimization on the final design. Thus, a fixed finite element mesh has been used, with a suitable order of approximation, to obtain the results. The effect of the

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discretization error on the final optimal design was not studied. The early work which involves the study of effect of discretization error, on the final optimum shape, was seen due to Kikuchi et al. [17]. In the shape optimization procedure, due to change in shape, the mesh gets distorted and the final design is sensitive to the approximation error associated with the discretization. Thus, improvement in the quality of the approximation is needed [17]. In this work global error estimates were developed for adaptive refinement strategies. A similar work was carried out by Hinton et al. [18]. Weck et al. [19] have worked on saving the computational cost during optimization of composite structures with ply-orientations and thickness as optimization variables using adaptive meshing technique. Benneet and Botkin [20] aimed to provide more accurate estimate of the true optimal solution with the effect of adaptive meshing on stresses used in constraint evaluation. Schleupen et al. [21] developed both global (based on error of the strain energy of overall structure) and local error estimates (based on error in a particular quantity of interest like displacement or stress component) for global and local adaptive refinements separately. The potential of these two techniques were then compared through two dimensional shape optimization problems. Morin et al. [22] developed an algorithm based on adaptive finite element method to equidistribute the errors due to shape optimization and discretization to optimize the computational cost. An application to X-FEM based structural optimization can be seen in [23]. An evolutionary technique was used along with sensitivity analysis, for a low cost adaptive remeshing, in shape optimization problems by Bugeda et al. [24].

The application of adaptive meshing using goal oriented error control for topology optimization was done by Bruggi and Verani [25]. Another application of adaptive refinement approach to topology optimization can be seen in Wang et al. [26].

In the present work a design of laminated composite plate, with a centrally located cutout for minimum weight, subjected to a constraint that the plate should not fail under first-ply failure load criterion, has been studied. Here, an attempt is made to demonstrate the effect of reliability of constraints on the final optimal solution. Initially, the final optimal solution is obtained without considering reliability of the computed data used in the evaluation of first-ply failure load constraint. The process is then repeated with a control on the reliability of the computed data, i.e. effect of discretization error control on final optimal shape. In the present work a higher order shear deformable plate theory proposed by Reddy [27] has been adopted and implemented in a finite element code. Further, Zienkiewicz-Zhu (ZZ) [28–31] type a-posteriori patch recovery based error estimator is developed for strain field corresponding to the plate model considered. Although, the use of Genetic Algorithms (GA) (for example, [12]) and evolutionary algorithms (for example [24]) is very popular in optimization studies, in the present study we have used a conventional optimization algorithm - Complex Search [32] to obtain an optimal design. Finally, the effect of evaluation of first-ply failure load constraint, with and without control in discretization error, is studied. Here, the Tsai-Wu first-ply failure criterion [33] has been used as a constraint.

2. Problem formulation

In this section a higher order shear deformable plate theory due to Reddy [27] is presented followed by the finite element formulation for this plate model.

2.1. Higher order plate model

Symmetric laminates find many applications in the aircraft industry. Although symmetric laminates are simple to analyze and design, some specific applications of laminated composites require unsymmetric laminates. For example, the coupling between bending and extension exhibited by this type of laminates is an essential feature of jet turbine fan blades with pre-twist. It can be noted that the theories for unsymmetric laminates are applicable to symmetric laminates as a special case. Unlike symmetric laminates, unsymmetric laminates exhibit membrane-flexure coupling phenomenon, which necessitates the use of a displacement field containing both, membrane as well as flexure deformation terms which contribute to the overall response of a laminate. The analysis of laminated plates is based on the choice of a plate theory. Several plate theories have been developed with assumed variation of the displacement field in the transverse direction. For example, see plate theories in [3,4,12,34–38]. These theories attempt to give a higher order representation of strains in the laminate thickness direction. In the following, we present the details of plate theory due to Reddy [27] implemented in the present study. The displacement field

$$\mathbf{u}(x,y,z) = [u(x,y,z)\; v(x,y,z)\; w(x,y,z)]^T \tag{1}$$

is derived from the expanded Taylor's series in terms of thickness coordinate z. Here, $u(x,y,z)$, $v(x,y,z)$ and $w(x,y,z)$ are the displacement components along x, y and z axes, respectively. These components, following the work of Reddy [27], are given as

$$\begin{aligned} u(x,y,z) &= u_0(x,y) + z\theta_x(x,y) + z^2\phi_x(x,y) + z^3\psi_x(x,y) \\ v(x,y,z) &= v_0(x,y) + z\theta_y(x,y) + z^2\phi_y(x,y) + z^3\psi_y(x,y) \\ w(x,y,z) &= w_0(x,y) \end{aligned} \tag{2}$$

In the expansion in Eq. (2), it is assumed that transverse normal strain ϵ_{zz} is zero. The linear strain–displacement relationships using small deformation theory can be obtained from this equation.

The condition that the transverse shear stresses vanish on the plate's top and bottom faces (see Fig. 1) is equivalent to the requirement that the corresponding strains be zero on these surfaces, i.e.

$$\gamma_{yz}\left(x,y,\pm\frac{d}{2}\right) = \gamma_{xz}\left(x,y,\pm\frac{d}{2}\right) = 0 \tag{3}$$

On introduction of these conditions in the expressions for transverse shear strains, the following relations are obtained.

$$\phi_x = \phi_y = 0; \quad \psi_y = -\frac{4}{3d^2}(\theta_y + w_{0,y}) \quad \text{and} \quad \psi_x = -\frac{4}{3d^2}(\theta_x + w_{0,x}) \tag{4}$$

The displacement field of Eq. (2) is modified by setting ϕ_x and ϕ_y to be zero according to conditions of Eq. (4). The resulting displacement field is now written as

$$\begin{aligned} u(x,y,z) &= u_0(x,y) + z\,\theta_x(x,y) + z^3\,\psi_x(x,y) \\ v(x,y,z) &= v_0(x,y) + z\,\theta_y(x,y) + z^3\,\psi_y(x,y) \\ w(x,y,z) &= w_0(x,y) \end{aligned} \tag{5}$$

In Eq. (5) u_0, v_0 and w_0 are the mid-plane displacement components while θ_x and θ_y are the rotations about y and x axes, respectively. Further, ψ_x and ψ_y are higher order terms in the Taylor's series expansion and are also defined at the mid-plane. Thus, the generalized displacement vector $\{\delta\}$ of the mid-surface contains seven degrees of freedom (DOF) and is given by:

$$\{\delta\} = \{u_0\; v_0\; w_0\; \theta_x\; \theta_y\; \psi_x\; \psi_y\}^T \tag{6}$$

The corresponding strain–displacement relations, using infinitesimal strains, are: