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# Time domain linear sampling method for qualitative identification of buried cavities from elastodynamic over-determined boundary data

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ABSTRACT

A time domain version of linear sampling method (LSM) is developed for elastic wave imaging of media including scatterers with arbitrary geometries. The LSM is an effective approach to image the geometrical features of unknown targets from multi-view data collected from measurement of casual waves. This study emphasizes the exploitation of the LSM using spectral finite element method (SFEM). A comprehensive set of numerical simulations on two-dimensional elastodynamic problems is presented to highlight many efficient features of the proposed fast qualitative LSM identification method such as its ability to locate an inclusion (e.g., a crack) and estimate its dimensions.



## 1. Introduction

Reconstruction of inclusions (e.g., defects and obstacles) in a non-accessible region from measures of the backscattered elastic waves is of much interest in various disciplines. For this purpose, the location, shape, and size of inclusions are identified for a wide range of problems in seismology, nondestructive evaluation, medical diagnosis, geophysics, and submarine detection. These problems are generally known in the literature as the 'inverse scattering' problems. In many practical fields such as structures, building components industries, and aerospace, numerical analysis and computational simulations are intensively replacing full-scale and prototype laboratory testing. Among various applications, crack detection at both manufacturing stage and during operating life of structural components is important for deciding about their repair or replacement. Practical observations may be generally useful for crack identification; however, for critical and full-scale structures (e.g., railway tracks, slab deck bridges, and aerospace structures), visual inspection may be difficult in practice. Identification of such defects requires over-determined data provided by a set of measurements. A comprehensive review in condition monitoring with particular emphasis on structural engineering applications was presented by Carden and Fanning [1]. Fan and Qiao [2] also reported a complete review on damage detection methods for beam- or plate-type structures. Kessler et al. [3] studied some damage detection procedures for in-situ damage detection of composite materials. In their numerical work they found an accurate and easy algorithm to determine time of flight (TOF) of a Lamb wave pulse between an actuator and sensor. Su et al. [4] extended Kessler's approach to a two-dimensional (2D) domain. In an experimental investigation, they [4] used four piezoelectric sensors located near to the vertices of a rectangular CF/EP plate. A nonlinear set of equations was obtained using TOFs and the solution of nonlinear equations yielded the coordinates of a point as crack center. The disadvantage of this method is that just one point has been shown as the crack location, which means the damage severity is unknown. Time reversal imaging is another Lamb wave based damage detection algorithm, which uses a concept similar to that of TOFs [5]. The fundamental concept in this method is that for any waves radiating from a source, which are subsequently scattered, reflected and refracted by scatterers, there exists a set of waves that can precisely retrace all the paths and converge in synchrony at the original source, as if time were going backwards [6]. Multiple solutions, complexity and low resolution results are the main shortcomings of this approach. There are many studies on the interaction of elastic/electromagnetic waves with unknown scatterers such as cracks. Lu et al. [7] studied the interaction of Lamb wave modes at varying frequencies with a through-thickness crack of different lengths in aluminum plates in terms of numerical and experimental studies. Lee [8] considered an inverse scattering problem to recover the impedance function for an arbitrary crack from the far field pattern. Nichols et al. [9] described a population-based Markov Chain Monte Carlo approach

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for efficient sampling of the damage parameter posterior distributions. Their approach estimated the state of damage in a cracked plate structure using simulated, free-decay response data.

Time delay of arrival (TDOA) method, from measurements provided by an array of sensors, has played an important role in electronics engineering, robotics, aerospace engineering, defense technologies and seismology for localizing radiating sources. TDOA-based localization has been often used for source localization and tracking [10]. In the context of elastodynamic, Noureini and Khaji [11] used the concept of TDOA-based localization to detect a crack in a plate. The proposed method is based on the scattering of elastic waves due to unknown scatterer.

In recent years, many research attempts led to the development of effective approaches for damage detection in structural components. Notwithstanding a significant progress was gained in numerical solutions of these problems during the last decades, several treatments of inverse scattering problems are genuinely nonlinear. The mathematical efforts made during the last decades to overcome this additional difficulty have made advancements in three categories of procedures: non-linear minimization-based approaches, linearization methods, and qualitative methods. Non-linear minimization-based approaches [12,13] are based on the minimization of a misfit cost function through iteratively solving the underlying forward scattering elastodynamic problem. Although such techniques can produce very precise reconstructions, they impose remarkable high computational cost associated with the quite accurate initial guess. Linearization methods [14,15] that are based on a weak scattering approximation (e.g., the Born approximation) are often limited by physical configurations. These methods depend crucially on the weak-scattering assumption. During the past two decades, the above-mentioned limitations have led to the conceptually distinct class of inverse scattering solutions, termed as "qualitative methods" for non-iterative obstacle reconstruction from far/near measurements of the scattered field [16]. These methods provide an effective alternative to the classic optimization approaches. These techniques can be classified as probe or sampling methods such as linear sampling method (LSM) [17,18], topological sensitivity (TS) [19,20], the probe method [21], and point source method [22]. In this regard, the LSM and the FM introduced in the inverse scattering literature for far-field acoustics are particularly attractive. This is due to the abilities of mentioned methods to provide accurate reconstruction of the location and shape of the unknown scatterer from measurements of near- or far-field patterns by simply observing the behavior of the norm of the regularized solution. This norm is bounded inside the targets and unbounded elsewhere. Moreover, the most interesting feature of these qualitative methods is that they do not require *a priori* information/assumption on the scatterer and/or the investigation domain. Furthermore, these methods have relatively low computational cost and can be applied to various types of defects including not-convex and not-connected ones. These methods have been applied in different applications, ranging from seismology, geophysics or submarine detection to non-destructive evaluation (NDE) and medical diagnosis. In the context of elastic waves, interior transmission problem has been investigated by introduction of the LSM for far-field problems [23].

Although the LSM has gained remarkable attention in inverse scattering theory dealing with wave patterns in the frequency domain, little attention has been devoted to its application for near-field elastic wave forms in the time domain. In most cases, measurements of scattered fields are based on a single frequency as input data to identify the unknown scatterers. Few extensions exist to multi-frequency data [24] and time-dependent measurements [25]. Chen et al. [25] developed the time domain version of the LSM for the scalar wave equation, and found that this version of the LSM satisfactorily provides reconstructions with a limited number of receivers/transmitters.

This paper addresses defect identification in elastic solids by means of the LSM in the context of 2D time domain elastodynamics. To the authors' best knowledge, this paper presents the first comprehensive numerical study of time domain version of linear sampling method (TDLSM) to identify defect in finite media such as plates. This paper is organized as follows. The direct scattering problems of interest are reviewed in Section 2, in which the spectral finite element method (SFEM) is introduced as the forward solution tool. The inverse scattering problem is presented in Section 3. In Section 4, the LSM formulation is re-defined and established for the underlying problem. Finally, the results of numerical examples are presented and discussed in Section 5.

## 2. Direct scattering problem

Let $\Omega \subset \mathbb{R}^d$ ($d = 2$ or $d = 3$) denote a linearly elastic body which occupies a closed and bounded domain with external surface $S$. This domain, which is referred as the reference body, is characterized by the Lame's constants $\lambda$, $\mu$, and mass density $\rho$. A bounded cavity with the closed traction-free boundary $\partial D$ is embedded in a homogeneous elastic media $\Omega$. The boundary $S$, which is identical for the reference domain $\Omega$ and the cavitated domain $\Omega(D) = \Omega \setminus D$, is decomposed into two portions: $S = S_N \cup S_D$, such that $S_N \cap S_D = \emptyset$ with $\bar{\boldsymbol{u}}(\boldsymbol{x},t)$ and $\bar{\boldsymbol{t}}(\boldsymbol{x},t)$ being the prescribed displacement and traction, respectively (see Fig. 1). Accordingly, the displacement field of a point $\tilde{\boldsymbol{x}} = [\tilde{x}_1, \tilde{x}_2]^T$ arises in $\Omega(D)$ at the time $t \in [0,T]$ is denoted by $\boldsymbol{u}^{\boldsymbol{D}}(\tilde{\boldsymbol{x}},t) = \left[u_1^{\boldsymbol{D}}(\tilde{\boldsymbol{x}},t), u_2^{\boldsymbol{D}}(\tilde{\boldsymbol{x}},t)\right]^T$. The displacement field $\boldsymbol{u}^{\boldsymbol{D}}(\tilde{\boldsymbol{x}},t)$ satisfies the well-known equation of motion as [26,27]

$$
\begin{aligned}
&\mathcal{L}\boldsymbol{u}^{\boldsymbol{D}}(\tilde{\boldsymbol{x}},t) = \boldsymbol{0}, \quad (\tilde{\boldsymbol{x}} \in \Omega(D), t \geqslant 0)\\
&\boldsymbol{t}(\boldsymbol{x},t) = \boldsymbol{0}, \quad (\boldsymbol{x} \in \partial D, t \geqslant 0)\\
&\boldsymbol{t}(\boldsymbol{x},t) = \bar{\boldsymbol{t}}(\boldsymbol{x},t), \quad (\boldsymbol{x} \in S_N, t \geqslant 0)\\
&\boldsymbol{u}^{\boldsymbol{D}}(\boldsymbol{x},t) = \bar{\boldsymbol{u}}(\boldsymbol{x},t), \quad (\boldsymbol{x} \in S_D, t \geqslant 0)\\
&\boldsymbol{u}^{\boldsymbol{D}}(\tilde{\boldsymbol{x}},0) = \dot{\boldsymbol{u}}^{\boldsymbol{D}}(\tilde{\boldsymbol{x}},0) = \boldsymbol{0}, \quad (\tilde{\boldsymbol{x}} \in \Omega(D))
\end{aligned}
\tag{1}
$$

where $\mathcal{L}$ denotes the Lamé–Navier partial differential operator defined by

$$
\mathcal{L}\boldsymbol{u}(\tilde{\boldsymbol{x}},t) = \mathrm{div}[\boldsymbol{\mathcal{C}} : \nabla \boldsymbol{u}(\tilde{\boldsymbol{x}},t)] - \rho \ddot{\boldsymbol{u}}(\tilde{\boldsymbol{x}},t) \tag{2}
$$

in which $\boldsymbol{\mathcal{C}}$ is the fourth-order elasticity tensor whose components include all parameters required to characterize the material properties. For the case of isotropic materials, the constant components $\mathcal{C}_{ijkl}$ may be expressed as

$$
\mathcal{C}_{ijkl} = \lambda\delta_{ij}\delta_{kl} + \mu(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}) \tag{3}
$$

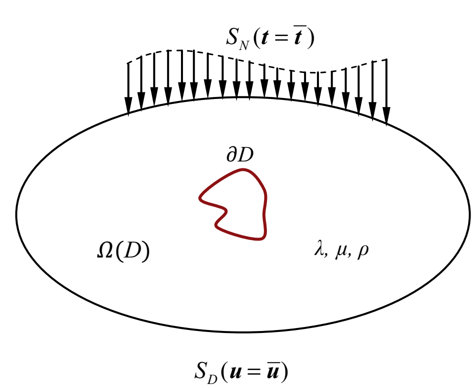


**Fig. 1.** A sample 2D domain ($\Omega$) with an embedded cavity.