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An unconstrained integral approximation of large sliding frictional contact between deformable solids

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ABSTRACT

A new integral approximation of frictional contact problems under large deformations is presented. Impenetrability, friction and the relevant complementarity conditions are expressed through a non-smooth equation, considered in the continuous setting. A weak formulation of this non-smooth complementarity equation is discretized through a standard Galerkin procedure, is linearized consistently and incorporated in a generalized Newton solution process. The resulting integral handling of contact and friction complementarity conditions, previously implemented for small deformations only, is extended in the present paper to large deformations. In total, the proposed method is relatively simple to implement, while its robustness is illustrated through numerical examples.

1. Introduction

Numerical modeling of frictional contact between solids undergoing large deformations is a challenging task, mainly because it involves complex geometrical and mechanical quantities that depend on an a priori unknown mapping between the surfaces in contact. Despite the multitude of very elaborate methods, proposed for solving this problem, there is an ongoing effort for improving the performance and robustness of currently available algorithms but also for simplifying the corresponding software implementations.

Although an exhaustive review of the field would be difficult, most methods that can represent frictional contact between deformable bodies under large deformations fall under the following categories:

(1) Node-to-segment methods [18], possibly enhanced with smoothing techniques [23].
(2) Mortar methods [10], possibly in combination with definition of contact segments [24].
(3) Contact domain methods [12,21,32] and intermediate surface methods [19].

Node-to-segment and mortar methods normally represent asymmetric formulations, in the sense that the surfaces in contact are treated differently by distinguishing between a master (or mortar or target) and a slave (or non-mortar or contactor) surface. On the contrary, contact domain and intermediate surface methods are by their nature symmetric, hence they are intrinsically applicable to cases like self-contact and simultaneous contact between more than two solids, where asymmetric formulations usually require special treatment. Their main drawback is that for an arbitrary three-dimensional geometry, triangulation of a contact domain or definition of an intermediate surface can be complex or not even guaranteed. The here proposed approximation is comparable to the mortar methods presented in [10,28], in the sense that it relies on the available discretization of the slave surface for performing numerical integration of all relevant contact terms.

Especially in the context of large deformations, the mapping between the surfaces in contact is an important component in formulating a numerical approximation of the frictional contact problem. Node-to-segment methods traditionally map points of the slave surface to their closest point projection onto the master surface. Hence, the master surface normals govern the definition of a gap function and its kinematics, presented in detail in [18]. This classical mapping will in the following be simply referred to as the projection strategy. A different approach for defining a mapping between the slave and master surfaces is to find the closest intersection with the master surface along the slave surface normals. This mapping which is more common for mortar methods, will in the following be referred to as the ray-tracing strategy.

Specifically referring to mortar methods for contact under large deformations, the formulations presented for instance in [10,16] employ the classical projection approach, while [25,34,29,11]

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present formulations that rely on the ray-tracing strategy. Other occurrences of the ray-tracing strategy can be found in [29] as a contact search method as well as in connection to contact problems under small deformations, for instance, in the segment-to-segment approach presented in [34] and in the Nitsche formulation introduced in [31]. The approach followed in the present paper relies on the ray-tracing strategy for deriving a contact formulation not depending on the curvature of the master surface with an optimality system which is not discontinuous across mesh edges or vertices, without requiring any smoothing technique.

Another crucial component in the numerical treatment of contact problems is the method for enforcing contact and friction conditions. Apart from the classical penalty method with its well known accuracy limitations, alternative approaches introduce multipliers for dealing with inequality and complementarity constraints, for instance, in the context of an augmented Lagrangian or interior point formulation. The unknown displacement and multiplier fields can be determined iteratively based on different fixed point techniques [17], including the very popular Uzawa method proposed in [26]. Alternatively, the generalized (or semi-smooth) Newton algorithm can be applied to the full system of equations including both displacements and multipliers [1,25] or equivalently the solution can be based on a primal–dual active set strategy [15].

One implication related to transitioning from penalty to Lagrange multiplier based formulations in the context of mortar methods, concerns the discretization of complementarity conditions. As discussed in detail in [9], penalty formulations in mortar methods permit an integral enforcement of the contact complementarity condition by evaluating it at quadrature points. On the contrary, Lagrange multiplier based mortar methods evaluate contact and friction complementarity conditions with respect to nodal values of the Lagrange multiplier and weighted gap or slip values [9,11]. This kind of discrete enforcement of complementarity conditions has two important consequences:

(a) The Lagrange multiplier field can only be approximated through Lagrange elements, so that finite element nodal values can be used in the evaluation of complementarity conditions.
(b) Each finite element node of the Lagrange multiplier can be associated to either the active or the inactive set of a complementarity condition. Intermediate states cannot be approximated adequately even if the number of quadrature points is increased.

The main characteristic of the here proposed method is an integral approximation of the contact and friction complementarity conditions in the context of an augmented Lagrangian formulation. Impenetrability, Coulomb friction stress threshold and the corresponding complementarity conditions are expressed as a semi-smooth equation in the continuous space, incorporated in the weak formulation of the problem and discretized according to a standard Galerkin procedure. Very few occurrences of such an integral approach can be found in the computational contact mechanics literature. To the authors' knowledge, the fundamental idea of an integral enforcement of a complementarity equation, capturing all contact and friction conditions, was originally proposed in [8]. Nevertheless, the actual implementation included in [8] is a nodal one, while the implementation found in [21] relies on a quadrature-point-wise definition of the unknown Lagrange multipliers. In the current work, similar to [17,26], the Lagrange multiplier field is approximated on a finite element space and contact and friction conditions are enforced in a weak sense. Unlike Refs. [17,26] however, the present work is not limited to the small deformations setting.

The proposed method, apart from representing a mathematically rigorous approximation, has the advantage that it does not require to prescribe constraints on the discretized Lagrange multiplier any longer, like for instance negativity of the contact pressure and a Coulomb threshold on the friction stress. The full set of contact, friction and complementarity constraints are already included in the weak formulation. As an interesting consequence, the formulation is independent of the finite element methods chosen to approximate the displacements and Lagrange multiplier fields. This characteristic offers the possibility of combining the proposed formulation, in the future, with less common approximations than the classical Lagrange finite elements, like for instance C^1 continuous Hermite and enriched finite elements as well as isogeometric analysis approximations of contact under large deformations, like [6,27].

The paper is organized in nine sections. Following this introduction, Section 2 presents the basic problem setting and some notation conventions. Section 3 provides a comparison between the classical projection and ray-tracing strategies. Section 4 presents the weak formulation of the frictionless case along with some comments about discontinuities in the contexts of the projection and ray-tracing strategies. Sections 5 and 6 respectively describe the proposed weak formulation and finite element approximation for frictional contact, while Section 7 gives some implementation details. Section 8 presents numerical results and Section 9 concludes the paper.

2. Problem setting and notations

Let $\Omega \subset \mathbb{R}^d$ denote the reference configuration of a deformable solid in a space of dimension $d = 2$ or 3, where Ω may either be connected or consist of more than one connected components like for instance shown in Fig. 1. A deformed configuration of the considered solid can be defined through a transformation φ which maps any point X of the reference configuration to a new point x:

$$\varphi : \Omega \longrightarrow \mathbb{R}^d$$

$$X \mapsto x = \varphi(X),$$

and is often written in terms of the displacement u relatively to the reference configuration as:

$$\varphi(X) = X + u(X).$$

In the deformed configuration Ω_t, at time t, different portions of the boundary $\partial\Omega$ of Ω may come into contact and interact with each other. In order to express this interaction mathematically, it is convenient to consider part of $\partial\Omega$ as a slave (or contactor) surface Γ^S and some other part as master (or target) surface Γ^M.

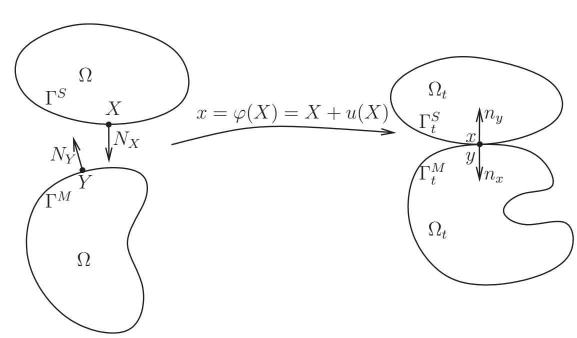

Fig. 1. Contact interface quantities in reference and deformed configurations.