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# A non-matching finite element-scaled boundary finite element coupled method for linear elastic crack propagation modelling

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ABSTRACT

A novel method coupling the scaled boundary finite element method (SBFEM) and the finite element method (FEM) is developed for linear elastic fracture modelling. A very simple but effective remeshing procedure based on the FE mesh only is used to accommodate crack propagation. The crack-tip region is modelled by an SBFE subdomain whose semi-analytical displacement solutions are used to extract accurate stress intensity factors. The SBFE subdomain is coupled with the surrounding FE mesh through virtual interfaces so that non-matching nodal discretisations of the shared boundaries can be used and only one SBFE subdomain is needed at a crack-tip. A few plane problems are modelled to validate the new method.



## 1. Introduction

There exist inherently various defects and flaws in engineering materials and structures from nano-, micro- to meso-scales. Under certain external loadings, these small-scale defects and flaws can develop into macro-scale cracks, whose initiation and propagation may severely affect the structural integrity and safety. Therefore, understanding crack propagation behaviour by laboratory experiments and numerical simulations has attracted tremendous attention in last five decades. This paper is focused on numerical modelling of crack propagation problems.

The difficulties and challenges of numerical modelling of crack propagation are reflected by numerous numerical methods developed so far, e.g., the finite element method (FEM), the boundary element method (BEM), the meshless or meshfree method, and more recently, the extended FEM (XFEM). The FEM is the most popular numerical method in simulating crack propagation because of the high generality and flexibility of the method in modelling structures with complex geometries, various boundaries and loading conditions [1–3]. However, when the FEM is used to simulate crack propagation, very fine crack-tip meshes or special elements are needed to calculate accurate stress intensity factors (SIFs) for crack propagation. This makes remeshing difficult. The BEM is another popular method that has been widely used to model fracture problems [4–7]. The modelled spatial dimensions are reduced by one because only boundaries are discretised, which makes remeshing much simpler. However, the BEM is applicable only to problems where Green's functions can be derived, which restricts its wide applicability [8]. The meshless or meshfree method [9] models a domain with boundaries and scattered nodes only. Nodal moving rather than remeshing around the crack-tip is carried out as the crack propagates. This method can calculate accurate SIFs but the computational cost is generally higher than the FEM [10]. In the XFEM, the displacement discontinuity across the crack faces is taken into account by adding discontinuous functions to the shape functions to avoid remeshing [11–14]. Extra terms of enrichment functions are needed to capture different crack-tip singularities. This makes the formulation of shape functions complicated and the numerical integration becomes involving and less accurate [14,15]. Many special crack-tip elements with built-in singularities, e.g. quarter point elements [16], hybrid Trefftz elements [17], hybrid crack elements (HCE) [18–20] have also been developed in the FEM, BEM and XFEM.

A more recent alternative is the scaled boundary finite element method (SBFEM). It is a semi-analytical method developed by Song and Wolf [21,22] in the late 1990s. This method not only combines the advantages of FEM and BEM but also exhibits additional advantages, e.g., it discretises boundaries only and the spatial dimension

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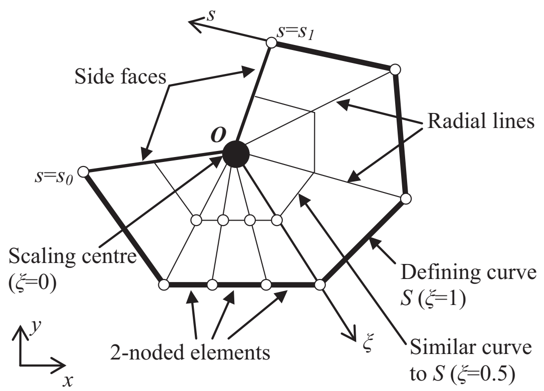


**Fig. 1.** A subdomain in SBFEM.

is reduced by one like the BEM, but requires no fundamental solutions; it avoids singular integrals and extends the BEM's applicability considerably [23–25]. It is very efficient in modelling problems with discontinuities and singularities because of its semi-analytical nature. Ooi and Yang [26,27] recently developed a hybrid FE–SBFE method capable of automatically modelling multiple crack propagation. The hybrid method retains the advantages of SBFEM such as its high accuracy in calculating SIFs and that of FEM such as its flexibility in modelling complicated geometries. However, several crack-tip SBFE subdomains are needed in the hybrid method to maintain the matching nodal discretisation with surrounding finite elements. New methods coupling BEM and SBFEM [28,29] and XFEM [30] have also been developed in computational fracture mechanics.

In the domain decomposition technique, a domain is divided by virtual interfaces into independent parts which can be meshed separately, and non-matching nodal discretisations can be used on the two sides of the virtual interfaces [31–34]. The displacement compatibility across the virtual interfaces can be ensured by using sufficiently high stiffness on the interfaces [35–37]. In fracture problems, a domain can be conveniently divided into two parts, the local crack-tip part modelled with fine meshes to calculate accurate SIFs, and the rest global part without cracks modelled with coarse meshes. Using the non-matching technique, only the local crack-tip part needs to be remeshed/refined as the crack propagates (e.g., [37–39]), whereas the global part often needs to be remeshed as well when the matching meshes are used [3,40].

This study proposes a non-matching SBFEM–FEM coupled method to simulate quasi-static crack propagation problems based on the linear elastic fracture mechanics (LEFM). In this method, a

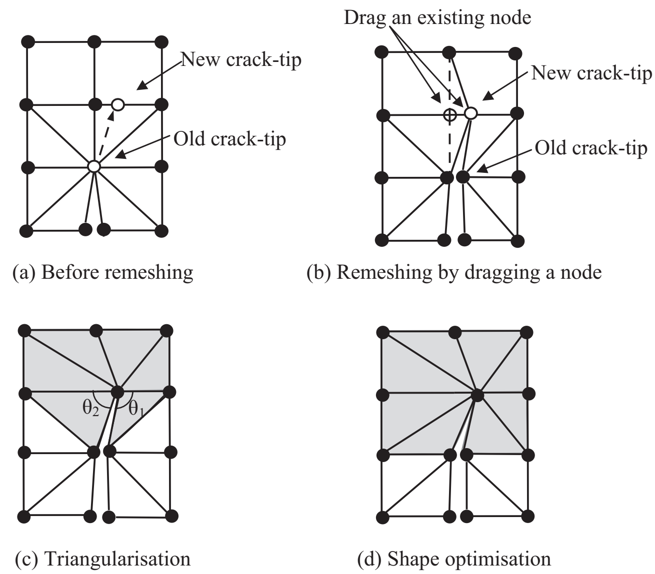


**Fig. 3.** FEM-based remeshing.

very simple remeshing procedure based on FE meshes only is used with accurate SIFs calculated by crack-tip SBFE subdomains. The main difference between the present method and the previous hybrid FE–SBFE method [26] is that in the present method, the SBFE subdomain boundary is coupled with the surrounding FE mesh boundary through virtual interfaces so that the nodal discretisations of the two boundaries can be different and only one SBFE subdomain is needed for one crack, whereas in the previous method, several subdomains are needed to maintain the matching nodal discretisation. Compared with the non-matching method based on FEM only, this method offers higher accuracy in calculating SIFs with much fewer degrees of freedom (DOF) due to the semi-analytical nature of SBFEM.

## 2. The non-matching SBFEM–FEM coupled method

### 2.1. The scaled boundary finite element method

In the SBFEM, a domain is divided into subdomains whose shapes and areas can be very different from one to another. Fig. 1 illustrates a two-dimensional (2D) subdomain, which is represented by scaling a defining curve $S$ relative to a scaling centre. A normalised radial coordinate $\xi$ is introduced, varying from zero at the scaling centre to unit on $S$. A circumferential coordinate $s$ is defined around the defining curve $S$. Thus, $\xi$ and $s$ form a local

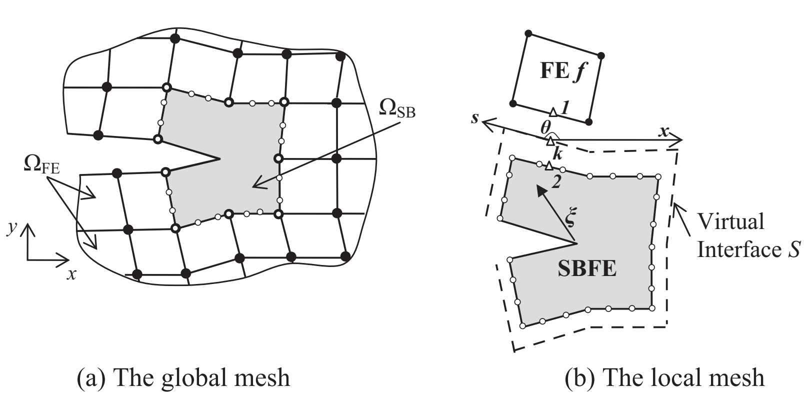


**Fig. 2.** Coupling SBFE and FE meshes.