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Stochastic response analysis of the scaled boundary finite element method and application to probabilistic fracture mechanics

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ABSTRACT

This paper proposes a stochastic response analysis method for the scaled boundary finite element method (SBFEM), through which the statistical characteristics of the structural responses subject to random uncertainty can be efficiently calculated. In the proposed method, an approximate approach is given to solve the first four statistical moments of the random responses of SBFEM. The probability density functions of the structural responses are calculated using the maximum entropy principle constrained by the calculated moments. The semi-analytical gradients of the responses with respect to the random variables are solved by developing an improved sensitivity analysis method of SBFEM. The proposed method is then applied to the structural reliability analysis and the probabilistic fracture mechanics analysis. Four numerical examples are investigated to demonstrate the validity of the proposed method.

1. Introduction

The scaled boundary finite-element method (SBFEM) is a semi-analytical method originally proposed by Song and Wolf [1,2]. The prominent feature of the SBFEM is that the whole problem domain is generated by scaling the boundary to a single point called the scaling center. Generally, it only requires meshing on the boundary of the analyzed domain reducing the spatial dimension of the problem by one in comparison to the finite element method (FEM) and does not need a fundamental solution in comparison to the boundary element method (BEM). For the analytical representation of the stress singularities in SBFEM, it has emerged as a promising technique for fracture analysis with stress intensity factors (SIFs) calculated accurately and efficiently.

The SBFEM received wide attention and have achieved great successes in the last two decades. Wolf [3] extended the SBFEM to calculate the response throughout the unbounded soil. Ekevid and Wiberg [4] analyzed the wave propagation related to three-dimensional high-speed train for unbounded domains based on the combination of conventional FEM and SBFEM. Tao et al. [5] solved the boundary-value problem composed of short-crested waves diffracted by a vertical circular cylinder using SBFEM, which was extended to deal with the interaction of water waves and porous offshore structure by Song and Tao [6]. Liu et al. [7,8] extended the SBFEM to solve short-crested wave interaction with a concentric structure with double-layer perforated cylinders. He et al. [9] developed an element-free Galerkin scaled boundary method for solving steady-state heat transfer problems. Song and Wolf [10] derived a semi-analytical solution of the singular stress occurring at cracks in anisotropic multi-materials by SBFEM. Song et al. [11] proposed a definition and evaluation procedure of generalized SIFs using SBFEM. Yang [12] and Ooi and Yang [13,14] used the SBFEM to analyze crack propagation problems. Li et al. [15] and Man et al. [16] presented a technique for structural analysis of piezoelectric materials based on SBFEM. Vu and Deeks [17] used fundamental solutions in the SBFEM to solve problems with concentrated loads. Gravenkamp et al. [18] presented an approach to compute dispersion curves of elastic waveguides based on SBFEM. Ooi et al. [19] developed a novel polygon-based SBFEM formulation, which was then applied to the analysis of non-linear elastic crack propagation [20], structural analysis of elasto-plastic materials [21] and fracture problem of functionally graded materials [22].

In the above-mentioned works, generally all the involved parameters such as material properties, loads and geometrical characteristics, were given specific values. Thus the whole structural response analysis was deterministic, namely deterministic input parameters cause the deterministic displacement, stress and strain responses. However, due to the effects of manufacturing and measuring errors as well as the unpredicted circumstance factors, many important parameters related to the material

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properties, loads and geometrical sizes of the structure often exist a certain degree of uncertainty. If we use the probability approach to deal with this uncertainty, an important stochastic response analysis problem then will be encountered. Through the stochastic response analysis, we can obtain the probabilistic distributions of the responses of the structure caused by the input parameters' uncertainty, which are then very important for the subsequent structural reliability analysis and safety design. Actually, research on the stochastic uncertain analysis for SBFEM has been conducted in few studies [23,24]. However, in these works which mainly focus on reliability analysis, the SBFEM was treated as a black-box solver, and the stochastic characteristics of the structural responses were achieved by calling a great number of evaluations of SBFEM, which generally leading to a relatively low computational efficiency. To develop an efficient stochastic response analysis algorithm then becomes very important for SBFEM, which will make the SBFEM playing a more important role in some important fields such as reliability analysis and probabilistic fracture mechanics.

In this paper, we aim to propose an efficient stochastic response analysis algorithm for the SBFEM, through which the probability distributions of the responses caused by the uncertain inputs can be efficiently obtained. The remainder of this paper is organized as follows. Section 2 summarizes the basic theory of SBFEM; Section 3 presents the formulation of the SBFEM-based stochastic response analysis method; Sections 4 and 5 give the applications of the method to structural reliability analysis and probabilistic fracture mechanics; Section 6 gives the numerical analysis, and Section 7 summarizes the conclusion of the paper.

2. Summary of the SBFEM

The fundamentals of the SBFEM are given in many publications (e.g., [1,2,25]) and the key equations are given below for the convenience of discussion. As depicted in Fig. 1, the SBFEM introduces a coordinate system (ξ, η) by scaling the domain boundary relative to a scaling center O: the radial coordinate ξ points to the boundary from the scaling center where its value is zero while its value is 1 on the boundary, and the circumferential coordinate η is along the boundary direction. The displacement field in the domain V is approximated analytically in the radial direction and in the FEM sense in the circumferential direction. The scaling center is chosen such that the whole boundary is visible from it. This can always be realized by dividing the domain into sub-domains. Suppose that the origin of the Cartesian coordinate system is selected at the scaling center. The scaled boundary and Cartesian coordinate systems are related by the scaling equations:

$$x = \xi x_\eta(\eta) = \xi \mathbf{N}(\eta)\{x\}, \quad y = \xi y_\eta(\eta) = \xi \mathbf{N}(\eta)\{y\} \tag{1}$$

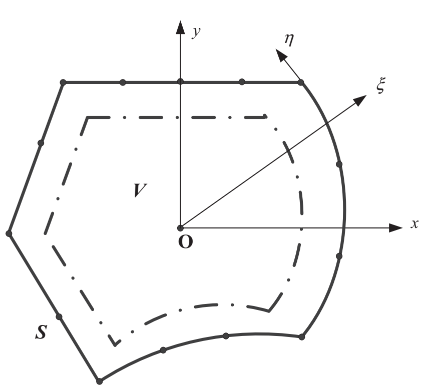

Fig. 1. Bounded domain in scaled boundary coordinates.

where $(x_\eta(\eta), y_\eta(\eta))$ describes the boundary coordinates using continuous piecewise smooth functions. $(\{x\}, \{y\})$ describes the boundary nodes' coordinates and $\mathbf{N}(\eta)$ is the shape function. The displacements are expressed as the following form:

$$\mathbf{u}(\xi, \eta) = \mathbf{N}^u(\eta)\mathbf{u}(\xi) \tag{2}$$

where $\mathbf{u}(\xi)$ are nodal displacement functions along the radial lines, $\mathbf{N}^u(\eta)$ is the shape function matrix at the circumferential direction. The whole domain displacements can be obtained by interpolating $\mathbf{u}(\xi)$ along the circumferential lines.

The scaled boundary finite element equations can be derived according to the virtual work principle in elastic statics [25]:

$$\mathbf{p} = \mathbf{E}_0 \frac{\partial \mathbf{u}(\xi)}{\partial \xi} + \mathbf{E}_1^T \mathbf{u}(\xi)\bigg|_{\xi=1} \tag{3}$$

$$\mathbf{E}_0 \xi^2 \frac{\partial^2 \mathbf{u}(\xi)}{\partial^2 \xi} + [\mathbf{E}_0 + \mathbf{E}_1^T - \mathbf{E}_1]\xi \frac{\partial \mathbf{u}(\xi)}{\partial \xi} - \mathbf{E}_2 \mathbf{u}(\xi) = 0 \tag{4}$$

where $\mathbf{p}$ can be identified as the equivalent nodal forces. The coefficient matrices $\mathbf{E}_0$, $\mathbf{E}_1$ and $\mathbf{E}_2$ are integrated along the boundary S:

$$\mathbf{E}_0 = \int_S \mathbf{B}_1^T(\eta)\mathbf{D}\mathbf{B}_1(\eta)|J|d\eta \tag{5}$$

$$\mathbf{E}_1 = \int_S \mathbf{B}_2^T(\eta)\mathbf{D}\mathbf{B}_1(\eta)|J|d\eta \tag{6}$$

$$\mathbf{E}_2 = \int_S \mathbf{B}_2^T(\eta)\mathbf{D}\mathbf{B}_2(\eta)|J|d\eta \tag{7}$$

where $\mathbf{B}_1(\eta)$ and $\mathbf{B}_2(\eta)$ describe the strain–displacement relationship with:

$$\mathbf{B}_1(\eta) = \frac{1}{|J|}\left[\mathbf{L}_1 \frac{\partial y_\eta(\eta)}{\partial \eta} - \mathbf{L}_2 \frac{\partial x_\eta(\eta)}{\partial \eta}\right]\mathbf{N}^u(\eta) \tag{8}$$

$$\mathbf{B}_2(\eta) = \frac{1}{|J|}\left[-\mathbf{L}_1 y_\eta(\eta) + \mathbf{L}_2 x_\eta(\eta)\right]\frac{\partial \mathbf{N}^u(\eta)}{\partial \eta} \tag{9}$$

where $\mathbf{L}_1$ and $\mathbf{L}_2$ are two constant matrices, which can be obtained by mapping the linear operator to the SBFEM coordinate system. $|J|$ is the determinant of the Jacobian matrix on the boundary:

$$|J| = x_\eta(\eta)\frac{\partial y_\eta(\eta)}{\partial \eta} - y_\eta(\eta)\frac{x_\eta(\eta)}{\partial \eta} \tag{10}$$

The solutions of the radial displacement have the following form:

$$\mathbf{u}(\xi) = \sum_{i=1}^{n} c_i \xi^{-\lambda_i} \boldsymbol{\phi}_i \tag{11}$$

where c_i are the integration constant, the exponent λ_i and corresponding vector $\boldsymbol{\phi}_i$ can be interpreted as independent deformation mode, and n is the total degrees of freedom of the nodes. The displacements for each mode take the form $u(\xi) = \xi^{-\lambda}\boldsymbol{\phi}$. Substituting this solution into the scaled boundary finite element equations and assembling together the two sets of equations leads to a linear eigenproblem:

$$\mathbf{Z}\boldsymbol{\Phi} = \boldsymbol{\Phi}\boldsymbol{\lambda}, \quad Z = \begin{bmatrix} \mathbf{Z}_{11} & \mathbf{Z}_{12} \\ \mathbf{Z}_{21} & \mathbf{Z}_{22} \end{bmatrix} = \begin{bmatrix} \mathbf{E}_0^{-1}\mathbf{E}_1^T & -\mathbf{E}_0^{-1} \\ \mathbf{E}_1\mathbf{E}_0^{-1}\mathbf{E}_1^T - \mathbf{E}_2 & -\mathbf{E}_1\mathbf{E}_0^{-1} \end{bmatrix} \tag{12}$$

where $\mathbf{Z}$ is a $2n \times 2n$ Hamilton matrix. There are $2n$ modes in the solution of this standard eigenproblem, and for a bounded domain only the modes with non-positive real components of eigenvalues $(\lambda = [\lambda_1, \lambda_2, \ldots, \lambda_n])$ lead to finite displacements at the scaling center. Therefore, the eigenvalues and eigenvectors (Eq. (12)) are partitioned as follows: