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A Lagrangian finite element approach for the simulation of water-waves induced by landslides

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ABSTRACT

In the present investigation we apply a pure Lagrangian finite element method to simulate nonlinear water waves due to landslides, of interest to ocean, coastal as well as to dam engineers. Validation is carried out by comparison between the computed prediction and experimental data of water waves generated by a two dimensional triangular rigid body and by a deformable granular mass sliding into water. The calculated results show that the simulations agree well with experimental observation illustrating the potential of the proposed scheme.

1. Introduction

Large scale movements of fluids and solids play a key role in many problems of environmental sciences [1] which include, but are not limited to, tsunamis [2], landslides [3], volcanic lava flows [4,5], snow and debris avalanche [6], dam breaks [7,8].

In particular specific attention has always been devoted by ocean, coastal and dam engineers [9,10] to landslides impacting water at high speed and inducing large waves. These small-scale tsunamis are characterized by short wave lengths and high amplitude and can produce impressive run-up heights at the water basin borders.

Such phenomena can be triggered by exceptional natural hazards associated to erosion, fault movements and intense earthquake vibrations, storms, heavy rainfalls or water level fluctuations [11].

Numerous instances of similar events have been observed all over the world [11]. Near coastal regions, the largest waves created by landslides occurred in Alaska (Lituya Bay and Disenchantment Bay), in Japan (Shimabara Bay), in French Polynesia, and in many Norwegian fjords. In steep mountainous regions, large water waves were generated by rockslides into reservoirs, lakes, or rivers. For instance, in October 1963 the southern flank of Mount Toc (Italy) collapsed and the partially submerged rockslide penetrated into the Vajont reservoir at velocities of up to 30 m/s. Wave run-up reached a highest level of 245 m above dam crest. In May 1981 a lobe from an avalanche off Mount St. Helens rammed into Spirit Lake at an assumed velocity of 70 to 80 m/s. The impact caused a wave run-up of 260 m above original lake level.

The catasprophic relevance of these phenomena has stimulated intensive investigations in order to assess the risk associated to the magnitude of flooding in neighboring areas. However, due to the complexity of comprehensive mathematical models, engineering predictions initially relied on experimental tests which are now currently employed as benchmarks for numerical simulations.

In [12,9] water waves were generated in a constant depth channel by allowing rigid bodies to slide down an inclined plane. In [13] water wave heights resulting from horizontal motion of a sloping incline in a constant depth channel were measured. Similar experiments with rigid bodies of different shapes have been reported by [14–16].

More recently the effect of deformable landslides impacting water basins has been analysed. In [10] a mass of sand with varying grain sizes was allowed to slide freely down a frictionless inclined plane. In [11,17] a pneumatic landslide generator was developed to control the physical impact characteristics of granular landslides accelerated down a hillslope ramp of 45°.

However, to help understanding the nature of the processes involved and predicting the detailed outcomes in specific locations with possibly complicated geomorphology, the development of computational tools for the modeling of water waves induced by landslides seems to be unavoidable.

The scope of this contribution is limited to those problems where also the landslide can be described as a continuum, possibly non-linear, viscous fluid. Even if this is clearly a restrictive assumption, according to [10] many materials such as snow, volcanic lava, submarine, and river sediments behave approximately like a

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non-Newtonian fluid which might be reasonably described e.g., by means of the Bingham model. These materials display a critical value of shear stress below which they behave like a rigid body. Above this threshold, the material behaves like a Newtonian fluid. When this assumption is not realistic, e.g., in the case of granular particulate solids where the micromechanical or particle level simulation is required, specific tools like the discrete-element modelling (DEM) should be employed, but are not addressed herein [1].

Even if we restrict our attention to continuum models for landslides, the numerical analysis of such phenomena implies the ability to track interfaces and free surfaces undergoing large displacements, to simulate mixing of different components, to account for complex constitutive behaviours and multiphysics phenomena. These applications call for the development of robust and research demanding general-purpose predictive tools. For this reason, empirical laws based on experiments are still in use [18].

Initially [19,20], the numerical approaches for the simulation of rockfall- or landslide-generated waves were based on simplified models which solved the depth-averaged Navier–Stokes equations for shallow water waves. However, neglecting vertical accelerations proves inaccurate in the generation zone where depth changes rapidly, and on the shore, where run-up and wave-breaking occur.

One of the first attempts to address the overall numerical problem in a general way was put forward in [9] where nonlinear, incompressible, two-dimensional flows with free surfaces were addressed by solving the complete Navier–Stokes equations with an Eulerian finite difference technique.

However, the issue of simulating evolving free-surfaces makes Eulerian approaches, where material flows through a fixed mesh in space, not well suited for the problem at hand. Indeed, dedicated algorithms, like the Volume of Fluid [21] or the Level Set [22] methods are required, but their application remains somewhat problematic.

Although several different alternative possibilities are in principle possible, Lagrangian approaches seem to be the most promising. Indeed, the motion of material particles is tracked, automatically capturing the free surface configuration and not limiting a priori in any way the movement of the analysed fluid. If the nodes of the mesh of a classical Finite Element Method (FEM) remain attached to material nodes and move along with them, one major disadvantage is that the mesh may distort severely and frequent time-consuming remeshing might be necessary, as discussed in the sequel of the paper.

Meshless Lagrangian approaches, like the Smoothed Particle Hydro-dynamics (SPH), avoid this issue by eliminating the need to mesh the domain. For this reason SPH has recently deserved considerable attention in this field [23,16,24].

In the review paper [1] several applications are presented to highlight the potentialities of SPH in the environmental sciences; in [25,26] 2D standard SPH analyses are compared with the results of several experiments on rigid and deformable landslides while SPH is employed in [27] to model 2D and 3D flow-like landslides. The classical formulation of SPH is enriched in [28] with a model for turbulence.

However these approaches are generally based on the strong form of equilibrium and conservation equations, which has sometimes attracted criticism. Whenever possible, it is commonly accepted that solving the equilibrium equations in a weak form enhances the accuracy, stability and robusteness of the numerical approach.

Very little attention has been devoted to assess the applicability of FEM Lagrangian approaches in this context, which, on the contrary, seem very promising. Hence, following some recent advances in the field of Lagrangian FEM and robust meshing algorithms, in this contribution we apply a fully Lagrangian approach with surface tracking capabilities based on a continuous re-triangulation of the domain, in the spirit of the so called Particle Finite Element Method (PFEM) [29]. The PFEM is a method for the solution of fluid-dynamics problems including free-surface flows and breaking waves [29], but also fluid–structure interactions [30] or fluid-object interactions [31]. This method has been applied to different engineering problems and has been also validated against experiments [32].

After recalling the basic governing equations in Section 2, we briefly review the concepts of the PFEM according to the specific formulation presented in [33] and present two applications to the simulation of waves generated by a rigid and a deformable landslide for which experimental data are available.

2. Governing equations

In this investigation we focus on problems where the landslide and the water basin can be modeled as a continuum, incompressible, non-homogeneous fluid (see e.g., [10]) filling the domain Ω_t.

The equations of motion can thus be written as:

$$\rho \frac{D\mathbf{u}}{Dt} = \mathrm{div}\boldsymbol{\sigma} + \rho \mathbf{b} \quad \text{in } \Omega_t \times (0,T) \tag{1}$$

$$\mathrm{div}\ \mathbf{u} = 0 \quad \text{in } \Omega_t \times (0,T) \tag{2}$$

where $\mathbf{u} = \mathbf{u}(\mathbf{x},t)$ is the velocity, $\boldsymbol{\sigma} = \boldsymbol{\sigma}(\mathbf{x},t)$ the Cauchy stress tensor, $\rho(\mathbf{x})$ is the fluid density, $\mathbf{b}(\mathbf{x},t)$ the external body forces, $D/(D\,t)$ denotes the total time derivative and div is the divergence operator computed with respect to the current configuration $\mathbf{x}$. The problem (1), (2) has to be supplemented with appropriate initial and boundary conditions and suitable constitutive equations.

The behaviour of materials usually involved in landslides can often be described with reasonable accuracy by a Newtonian constitutive law, whereby the deviatoric part $\boldsymbol{\tau}$ of the Cauchy stress ($\boldsymbol{\sigma} = -p\mathbf{I} + \boldsymbol{\tau}$, where $\mathbf{I}$ is the identity tensor and p denotes pressure) is related to the velocity gradient through the viscosity μ:

$$\boldsymbol{\tau} = 2\mu\boldsymbol{\epsilon}(\mathbf{u}) \tag{3}$$

where $\boldsymbol{\epsilon}(\mathbf{u}) = (1/2)(\mathrm{grad}\,\mathbf{u} + \mathrm{grad}\,\mathbf{u}^T)$ is the symmetric part of the velocity gradient. In many applications, however, the material of the landslide is better described as a non-Newtonian fluid according, e.g., to a classical Bingham law:

$$\boldsymbol{\tau} = 2\mu\boldsymbol{\epsilon}(\mathbf{u}) + \tau_0 \frac{\boldsymbol{\epsilon}(\mathbf{u})}{\|\boldsymbol{\epsilon}(\mathbf{u})\|} \quad \text{if } \|\boldsymbol{\tau}\| > \tau_0 \tag{4}$$

$$\boldsymbol{\epsilon}(\mathbf{u}) = 0 \quad \text{otherwise} \tag{5}$$

where $\|\cdot\|$ denotes the norms:

$$\|\boldsymbol{\epsilon}(\mathbf{u})\| = \sqrt{\frac{1}{2}\boldsymbol{\epsilon} : \boldsymbol{\epsilon}} \qquad \|\boldsymbol{\tau}\| = \sqrt{\frac{1}{2}\boldsymbol{\tau} : \boldsymbol{\tau}} \tag{6}$$

The coefficients in Eq. (6) have been defined so as to be consistent with the classical 1D Couette law

$$\tau_{xy} = \mu \frac{\mathrm{d}u_x}{\mathrm{d}y} + \tau_0\ \mathrm{sign}\left(\frac{\mathrm{d}u_x}{\mathrm{d}y}\right)$$

written for a couple of plates in the $x - y$ reference system, with the upper plate moving along x direction.

The piece-wise linear incremental behaviour implied by Eqs. (4) and (5) may produce numerical difficulties and consequently an approximation based on a smoothing of Eqs. (4), (5) is usually preferred. Here, following [34], a law based on an exponential evolution of the viscosity is adopted:

$$\boldsymbol{\tau} = 2\tilde{\mu}\boldsymbol{\epsilon}(\mathbf{u}) = \left[2\mu + \frac{\tau_0}{\|\boldsymbol{\epsilon}\|}\left(1 - e^{-n\|\boldsymbol{\epsilon}\|}\right)\right]\boldsymbol{\epsilon}(\mathbf{u}) \quad \forall \|\boldsymbol{\tau}\| \tag{7}$$