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Tests of non linear Gaussian term structure models

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ABSTRACT

Since the 2008 financial crisis Government bond yields in US, Europe and elsewhere have been historically low and challenged term structure models that cannot rule out negative yields. This paper uses US and German Government yields to test three factor Gaussian models that do and that do not rule out negative yields, namely affine models, quadratic models, extensions of the Black and Black–Karasinski models. Quadratic models and a Vasicek-type model best fit observed yields when the stochastic factors driving the short rate are correlated. However the Black–Karasinski model for the US and the Black model for both US and Germany can best fit yields when interest rates are lowest, i.e. after 2008, despite the restriction of independent factors driving the short rate. A new linear-quadratic model whereby the central tendency of the short rate is a non-negative quadratic function of Gaussian factors performs particularly well for German yields. All models fit German yields better than US yields. All models fit the one year yield worse than longer term yields.

1. Introduction

Since the 2008 financial crisis, due to strongly expansionary monetary policies, Government bond yields in US, Europe and elsewhere have been historically low almost to resemble Japanese Government bond yields. This setting challenges affine Gaussian term structure models (AGTSM) that do not rule out negative yields. Therefore this study uses US and German Government bond yields to test term structure models that do and do not rule out negative yields, and in particular models in which the instantaneous short interest rate is a non-negative function of Gaussian latent factors, such as quadratic term structure models (QTSM) and extensions of the Black (1995) and Black–Karasinski (1991) models. Black and Black–Karasinski models are hereafter referred to as BBKM.

AGTSM, QTSM and BBKM are all driven by Gaussian latent factors. Gaussian factors are tractable and do not suffer from the admissibility restrictions that affect more general affine stochastic differential equations. As a result market prices of risk can be freely specified and the pricing models better fit observed yields as explained in Dai and Singleton (2002) for the case of AGTSM. Hence this paper concentrates on Gaussian latent factors driving the short rate.

In AGTSM and QTSM the short interest rate is either an affine or quadratic function of the latent Gaussian factors. Instead in BBKM the short rate is a more general non-negative function of the latent Gaussian factors. QTSM and BBKM can rule out a negative short interest rate and negative yields, unlike AGTSM. BBKM require numerical solutions for bond valuation, which become burdensome when the latent factors are not independent, therefore this paper concentrates on three independent

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factors driving the short interest rate for BBKM. AGTSM and QTSM have closed-form or quasi-closed form solutions for bond prices even when the latent factors are not independent. Unlike in AGTSM, in QTSM and BBKM the conditional variance of yields increases with the level of yields; in this sense yields are "hetero-schedastic".

The main empirical finding is that quadratic models and a Vasicek-type affine model with non-independent factors generally fit US and German bond yields better than BBKM with independent factors. The "non-independence" of the factors generally seems more important than the "non-negativity" of model predicted yields over the sample period 1999–2011. However BBKM can best fit yields when central bank interest rates are lowest, especially after 2008, despite independence of the factors driving the short rate. When interest rates are lowest, a variant of the Black model fits US and German yields particularly well and the Black–Karasinski model fits US yields particularly well. The paper also tests a new linear-quadratic model whereby the short rate may turn negative, while the central tendency of the short rate is a quadratic non-negative function of Gaussian latent factors. This quadratic model fits German yields particularly well. All models fit German yields better than US yields. All models fit yields for short maturities of one or two years less well than yields of longer maturities.

The paper is organised as follows. The next section reviews the most relevant literature. Then two sections present the theoretical pricing models. Another section illustrates the empirical performance of the models. The conclusions follow.

2. Literature

The literature on dynamic term structure models is too vast to be summarised in this paper. A good survey is Dai and Singleton (2003). Here we refer only to Gaussian term structure models that simply rule out arbitrage and abstract from the macro-economy. Vasicek (1977) and Langetieg (1980) first studied affine Gaussian models. Babbs and Nowman (1999) used the Kalman Filter to estimate affine Gaussian models. Dai and Singleton (2002) tested general affine Gaussian models. Nowman (2010) successfully fitted a two factor affine Gaussian model to Euro and UK Sterling yield curves using the Kalman Filter. Joslin et al. (2011) studied affine Gaussian models whereby the factors are observable portfolios of yields. Duffee (2011) used affine Gaussian models to show that yields cannot detect variation in US Government bond risk-premia.

QTSM were studied already in the nineties and then in Ahn et al. (2002), Leippold and Wu (2002, 2003), and Chen et al. (2004). Ahn et al. (2002) illustrated the empirical advantage of general QTSM over affine models due to the unrestricted correlation between the factors driving the short interest rate. Gourieroux and Sufana (2005) and Realdon (2006, 2011) presented discrete time QTSM. Li and Zhao (2006) used a QTSM to provide evidence of un-spanned stochastic volatility in the pricing of interest rate derivatives.

Other notable Gaussian models outside the families of affine or quadratic Gaussian models are those of Black and Karasinski (1991), Black (1995). Using the Japanese term structure of interest rates, Realdon (2009) tested a two factor version of the Black model. Kim and Singleton (2012) tested various non-affine Gaussian term structure models with two latent factors using Japanese Government bond yields. Instead this paper tests multifactor versions of the affine Gaussian model, Black model, Black–Karasinski model and discrete time quadratic models using US and German Government bond yields.

3. Extended Black and Black–Karasinski models (BBKM)

The paper tests an extension of the Black (1995) model in which the time t default-free instantaneous short interest rate r_t is a function of the time t value of three latent factors $x_{1,t},\ x_{2,t},\ x_{3,t}$ so that

$$r_t = \sum_{i=1}^{3} \max\,(x_{i,t}, 0)^q. \quad (1)$$

q is a constant and will be set equal to 1 or 2. When $q = 1$ we have a three factor generalisation of Black's (1995) model. We consider the case where $q = 2$ because unreported tests show good empirical performance in comparison to other cases where q differs from 2. Given a filtered probability space with the usual properties, we assume

$$dx_{i,t} = \kappa_i(\mu_i - x_{i,t})dt + \sigma_i dw^{\mathbb{Q}}_{i,t}$$

for $i = 1,\ 2,\ 3$. $dx_{i,t}$ is the stochastic differential of the factor x_i and $dw^{\mathbb{Q}}_{i,t}$ the stochastic differential of a Wiener process in the risk-neutral measure $\mathbb{Q}$ over the infinitesimal time interval $[t, t + dt]$. The Wiener processes are independent unless otherwise stated, therefore $dw^{\mathbb{Q}}_{1,t}dw^{\mathbb{Q}}_{2,t} = dw^{\mathbb{Q}}_{1,t}dw^{\mathbb{Q}}_{3,t} = dw^{\mathbb{Q}}_{2,t}dw^{\mathbb{Q}}_{3,t} = 0$. $\kappa_i,\ \sigma_i,\ \mu_i$ for $i = 1,\ 2,\ 3$ are all constant parameters. Eq. (1) implies that r_t cannot turn negative and bond yields for maturities longer than the instantaneous maturity are guaranteed to be positive, even when $x_{i,t}$ are negative.

The paper also tests an extension of the Black–Karasinski (1991) model whereby $r_t = \sum_{i=1}^{3}\exp(x_{i,t})$ and a special case of the affine Gaussian model of Langetieg (1980) whereby $r_t = \sum_{i=1}^{3} x_{i,t}$. We refer to this version of the Langetieg model as the three factor Vasicek model.