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An empirical comparison of transformed diffusion models for VIX and VIX futures

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ABSTRACT

Transformed diffusions (TDs) are nonlinear functions of continuous-time affine diffusion processes. Since they are flexible models with tractable analytic properties, financial modelling with TDs has become increasing popular in recent years. We first provide a formal classification of TD models into drift-driven, diffusion-driven, and distribution-driven according to their empirical emphases and specification strategies. Motivated by the stylized distributional features of VIX such as skewness and excess kurtosis, we then propose a pair of new distribution-driven TDs for modelling VIX dynamics and pricing VIX futures by directly incorporating such information into the specification of the transformation. We conduct a comprehensive empirical investigation into the relative performance of the three classes of models against several empirically relevant criteria. Our focus is on the in-sample goodness-of-fit measure and the out-of-sample forecast accuracy for modelling VIX and pricing VIX futures, as well as the stock return predictability of the implied Variance Risk Premium. Our findings demonstrate that the newly proposed distribution-driven models have clear advantages over well-established alternatives in most of our exercises.

1. Introduction

As a measure of market volatility implied by traded S&P 500 index option prices, the Volatility Index (VIX), also known as the "investor fear gauge", has attracted enormous attention in recent years. For financial market participants, it is of the utmost importance to understand the dynamics of market volatility which is a crucial determinant of investment decisions. One of the most important strands of the literature focuses on the data generating process of VIX, since a realistic model for the VIX dynamics is vital for correct inference and accurate derivative pricing.

Continuous-time diffusion models are particularly useful for modelling financial variables, not only because they are flexible yet parsimonious analytical tools, but also because derivative pricing crucially relies on a "continuous-time" no-arbitrage argument (i.e. change of measure). For this reason, a growing number of studies have emerged on modelling the dynamics of VIX and price VIX futures and options using continuous-time models. Whaley (1993) used a Geometric Brownian Motion model which does not have the mean-reverting feature for pricing volatility futures contracts. Grunbichler and Longstaff (1996) considered pricing VIX futures and options assuming that VIX follows the Cox et al. (1985) square-root process (CIR). Detemple and Osakwe (2000) considered the log-normal Ornstein-Uhlenbeck (OU) model.

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Goard and Mazur (2013) advocated the so-called 3/2 model, which is the same as the Ahn and Gao (1999) model, for VIX and VIX options. More recently, Eraker and Wang (2015) proposed a new nonlinear diffusion model with a cubic drift term to study the Variance Risk Premium (VRP) implied by VIX futures prices.

One of the greatest challenges in diffusion modelling is to construct models that are sufficiently flexible to describe complex nonlinear dynamics in reality and sufficiently tractable to allow efficient inference such as the Maximum Likelihood (ML) and convenient and accurate derivative pricing. For this purpose, one useful approach is to consider transformation models in continuous time. Bu et al. (2011) promoted the idea of modelling nonlinear diffusion, say Y, as a transformation of a more tractable basic affine process, say X, for which the closed-form transition density is available. Prominent examples of tractable underlying diffusions include, but not restricted to, the OU and the CIR processes. For a given transformation, i.e. $Y = V(X)$, the transition density of the transformed diffusion (TD) Y is simply the distribution transformation of the transition density of X under mild conditions. Most importantly, if V is nonlinear and flexible, then Y would also be a nonlinear and conceivably more flexible diffusion process.

Financial modelling with TD models has been increasingly popular in recent years.[1] Bu et al. (2011) considered transformed OU and CIR processes with Constant Variance Elasticity (CEV) diffusion terms (denoted as OUCEV and CIRCEV) for modelling short-term interest rates.[2] They showed that their models can generate similar nonlinearities in both the drift and diffusion terms to those estimated nonparametrically (e.g. Ait-Sahalia, 1996a). More importantly, Bu et al. (2016b) showed that the CIRCEV model can provide much better fit to the VIX data than the Nonlinear Drift CEV (NLDCEV) model of Ait-Sahalia (1996b) and Conley et al. (1997). Detemple and Osakwe (2000) used the exponential transformation of the OU process (henceforth denoted as OUDO) for pricing volatility options. Goard and Mazur (2013) considered the reciprocal transformation of the CIR process for modelling VIX and pricing VIX options. More recently, Eraker and Wang (2015) considered a transformation of the CIR process which leads to a cubic drift function (henceforth denoted as CIREW). Using the Fourier transformation method, they derived a pricing formula for VIX futures and studied the VRP implied by the VIX futures prices.

The first contribution of this paper is to provide a formal classification of TDs in the literature, according to their empirical emphases and specification strategies. The first class are considered as "drift-driven" (e.g. CIREW and OUDO) where for an underlying process X, the transformation is derived so that the resulting TD Y has a desired drift function. The second class are "diffusion-driven" (e.g. OUCEV and CIRCEV) where the users specify a desired diffusion term of Y and the drift term is determined simultaneously. The third class are "distribution-driven" (e.g. OUFS) where the users specify the marginal distribution of Y to be a member a class of parametric distributions, based on which the transformation function is then derived. It can be shown that theoretically all three approaches are interchangeable since there is a intrinsic relationship between the drift, the diffusion and the marginal distribution of any stationary diffusion process. In practice, however, the empirical relevance and also the analytic tractability of the three strategies can be fairly different depending on the users' preferences.

Although TD models are increasingly popular in financial modelling, their full potential in modelling financial derivatives have not been fully explored. The second contribution of this paper is to propose a pair of new distribution-driven nonlinear TD processes purposefully designed to fully incorporate stylized features such as the skewness and excess kurtosis in the distribution of the VIX data while at the same time deliver a closed-form transition density for efficient likelihood inference and closed-form VIX futures pricing. Our new TDs (named as CIRSKST and OUSKST) are constructed as the transformed CIR and OU process, respectively, where we propose to use the Skewed Student-t distribution (SKST) of Hansen (1994) to directly explore the information in the marginal distribution of VIX. Following a simple argument of change of measure in continuous time, we derive a closed-form formula for the prices of the VIX futures contract.

As our third contribution of this paper, we provide a comprehensive empirical investigation into the comparative performance of the three classes of models for modelling time series of VIX data (i.e. under the physical P-measure) and for pricing VIX futures (i.e. under the risk-neutral Q-measure). Our data consist of 6352 daily observations of VIX from January 2, 1990 to March 20, 2015 and 19215 observations of VIX futures closing prices from March 26, 2004 to February 17, 2015. Our comparison is based on a set of empirically important criteria. Firstly, we compare the three classes of models in terms of their in-sample goodness-of-fit and out-of-sample forecasting accuracy for modelling the VIX dynamics under the physical measure. We then examine our competing models in terms of their in-sample and out-of-sample performance for pricing VIX futures, which is carried out jointly under both the physical and risk-neutral measures. Finally, following the arguments of Bollerslev et al. (2009) and Eraker and Wang (2015), we extract the time-varying VRP jointly inferred from the VIX and VIX futures data for each of our competing models and examine their predictability for S&P 500 index returns.

Our empirical analysis provides a number of important results. For modelling the VIX dynamics under the physical measure, the newly proposed distribution-driven models dominated well-established models in the literature in terms of both in-sample goodness-of-fit measures and out-of-sample forecast accuracy. While both the CIRSKST and the OUSKST models fitted the VIX data equally well, the latter produced the smallest average forecast errors. For modelling VIX and pricing VIX futures under the joint measures, very similar outcomes emerged. While the CIRSKST model dominated all other models in terms of in-sample performances, the distribution-driven class dominated the other two classes in terms of superior out-of-sample performances. In particular, the OUSKST model produced significantly smaller average forecast errors than any other competing model. All three classes of models performed rather similarly in terms of stock return predictabilities of the

[1] In a non-financial context, Forman and Sørensen (2014) considered a transformed OU process (henceforth denoted as OUFS) with a bimodal marginal distribution for modelling molecular dynamics.

[2] The CIRCEV model nests the Ahn and Gao (1999) and Goard and Mazur (2013) models as special cases.