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Shell finite element based on the Proper Generalized Decomposition for the modeling of cylindrical composite structures

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ABSTRACT

The introduction of the Proper Generalized Decomposition (PGD) is presented for the layer-wise modeling of heterogeneous cylindrical shells. The displacement field is approximated as a sum of separated functions of the in-plane coordinates and the transverse coordinate. This choice yields to an iterative process that consists of solving a 2D and 1D problem successively at each iteration. In the thickness direction, a fourth-order expansion in each layer is considered. For the in-plane description, classical Finite Element method is used. The approach is assessed through mechanical tests for thin/thick and deep/shallow laminated cylindrical shells. Both convergence rate and accuracy are discussed.

1. Introduction

Composite shells are widely used in the industrial field (aerospace, automotive, marine, medical industries...) due to their excellent mechanical properties, especially their high specific stiffness and strength. For composite design, accurate knowledge of displacements and stresses is required. One way consists in considering three-dimensional modelisation. However, due to the complexity of such numerical simulations, it is suitable to represent the problem as a two-dimensional model leading to the construction of shell theories. There are two ways for defining the approximation of the displacement field. A "pure shell model" can be considered in which the displacement is associated with the local curvilinear vectors, and strain and stress are deduced using differential geometry [1]. Alternatively, the shell-like solid approach [2] for obtaining shell FE is widely used in commercial software, as it is more simple. In this case, the displacement vector is defined in the global cartesian frame and jacobian matrix transformation is used to express strain and stress with respect to reference frame defined on the middle surface in order to introduce the constitutive law. In this approach, differentiation is simplified and the curvatures are not directly calculated [3]. So, the development of efficient computational models for the analysis of shells appears thus of major interest.

According to published research, various theories based on the Finite Element (FE) method for composite shells have been developed. In the following, most of the mentioned works refer to the pure shell model. Thus, two families of models [4] can be identified:

- the Equivalent Single Layer Models (ESLM), where the classical Shell Theory (CST/Koïter) and First Order Shear Deformation Theory (FSDT/Nagdhi) models can be found. The reader can refer to [9] to have a description of the assumptions on the strain for deriving different shell models. CST leads to inaccurate results for composites because both transverse shear and normal strains are neglected. Triangular and rectangular elements are used in [5–7] respectively, for shallow laminated shells. FSDT is the most popular model due to the possibility to use a C^0 FE, but it needs shear correction factors and transverse normal strain is always neglected. A rectangular isoparametric element based on a MITC approach is presented in a recent work [8]. So, Higher-order Shear Deformation Theories (HSDT) have been developed to overcome these drawbacks. Sgambitterra et al. [10] proposes a three-node flat shell element based on a 1,2-order theory including 7 parameters. A HSDT with 9 parameters based on a nine-node quadrilateral isoparametric element is developed by Kant and Menon [11]. A third-order theory with a four-node isoparametric element including 7 parameters is considered in [12]. Different theories based on the Carrera's Unified Formulation are addressed in [13].
In the ESLM context, a simple way to improve the estimation of the mechanical quantities consists in adding one zig-zag function (Murakami) in the expression of the displacement to introduce the slope discontinuity at the interface between two adjacent layers. It allows to describe the so-called zig-zag effect. It has been carried out by Brank [14]. The approach includes 7 parameters and is based on the Reissner's formulation. See also the work of Bhaskar [15] based on a HSDT approach.

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- the Layer-Wise Models (LWM) where the expression of the mechanical quantities is written in each layer. A quadratic triangle element based on a constant shear angle is considered in [16], but a shear correction factor is needed. A three-dimensional shell element is proposed in [17]. A LW triangle FE is developed in [18] with a condensation technique at the pre-processing level. [19] deals with a hybrid strain flat triangular FE based on the Hellinger–Reissner variational principle. Note that the transverse normal and shear stresses are only taken into account in [20] where a four-node isoparametric assumed strain is considered. We can mention the eight-node 3D hybrid-EAS solid shell element based on the Hu–Washizu variational principle in [21]. See also the previously mentioned work [13]. In all the aforementioned works, the number of unknowns depends on the number of layers.

As an alternative, refined models have been developed in order to improve the accuracy of ESL models avoiding the additional computational cost of LW approach. Based on physical considerations and after some algebraic transformations, the number of unknowns becomes independent of the number of layers. We can mention the recent work of Yasin [22] dedicated to shallow shells. A four-node quadrilateral element with 5 parameters is built. Shariyat [23] has also developed a so-called zig-zag model including 15 parameters. A full compatible Hermitian rectangular elements are employed. It should be also mentioned the work of Dau [1] where a C^1 triangular six-node FE (Argyris–Ganev) based on the Sinus model is considered involving 5 parameters. The approach ensures the continuity conditions of the transverse shear stresses at the interfaces between two adjacent layers.

For the present topics, it should be noted that the mentioned works are based on the Finite Element method for linear elasticity problem in mechanics and applied to laminated composites, knowing that many other approaches (meshless, analytical, semi-analytical...) are involved in open literature. Furthermore, the fundamental subject about the shear and membrane locking of shell is not addressed here. So, this above literature deals with only some aspects of the broad research activity about composite shells. An extensive assessment of different approaches for both various theories and/or finite element applications can be found in [24–31].

Over the past years, the Proper Generalized Decomposition (PGD) [32–35] has shown interesting features in the reduction model framework. It has been used in the context of separation of coordinate variables in multi-dimensional PDEs [34]. And in particular, it has been applied for composite beams and plates in [36–40].

The main goal of this work consists in assessing the Proper Generalized Decomposition to model cylindrical composite shell structures. So, the present approach is based on the separation representation where the displacements are written under the form of a sum of products of bidimensional polynomials of (ξ^1, ξ^2) and unidimensional polynomials of z. A piecewise fourth-order Lagrange polynomial of z is chosen. As far as the variation with respect to the in-plane coordinates is concerned, a 2D eight-node quadrilateral FE is employed. Using the PGD, each unknown function of (ξ^1, ξ^2) is classically approximated using one degree of freedom (dof) per node of the mesh and the LW unknown functions of z are global for the whole shell. Finally, the deduced non-linear problem implies the resolution of two linear problems alternatively. This process yields to a 2D and a 1D problems in which the number of unknowns is smaller than a classical Layerwise approach. The interesting feature of this approach lies on the possibility to have a higher-order z-expansion and to refine the description of the mechanical quantities through the thickness without increasing the computational cost. This is particularly suitable for the modeling of composite structures.

We now outline the remainder of this article. First, the shell definition and the differential geometry are recalled. Then, the mechanical formulation is given. The principles of the PGD are precised in the framework of our study. The particular assumption on the displacements yields a non-linear problem and an iterative process is chosen to solve this one. The FE discretization is also described and finally, numerical tests are performed. A preliminary convergence study is performed. Then, the influence of classical assumptions on the strains and the number of numerical layers are studied. The approach is also assessed for deep and shallow shells and different slenderness ratios. The accuracy of the results is evaluated by comparison with a 2D elasticity solution from [41].

2. Shell definitions and differential geometry

A shell $\mathcal{C}$ with a middle surface $\mathcal{S}$ and a constant thickness e, see Fig. 1, is defined by Bernadou [42]:

$$\mathcal{C} = \left\{ M \in \mathcal{R}^3 : \vec{OM}(\xi, \xi^3 = z) = \vec{\Phi}(\xi) + z\,\vec{a}_3;\ \xi \in \Omega;\ -\frac{1}{2}e \leqslant z \leqslant \frac{1}{2}e \right\}$$

where the middle surface can be described by a map $\vec{\Phi}$ from a parametric bidimensional domain Ω as:

$$\begin{aligned} \vec{\Phi} :\quad & \Omega \subset \mathcal{R}^2 \rightarrow \mathcal{S} \subset \mathcal{R}^3 \\ & \xi = (\xi^1, \xi^2) \mapsto \vec{\Phi}(\xi) \end{aligned} \tag{1}$$

In Fig. 1, the map $\vec{\Phi}$ describing the shell middle surface (in grey) and the local basis vectors are presented. The basis vectors $\vec{a}_i$ are defined for a point on $\mathcal{S}$ and the basis vectors $\vec{g}_i$ are defined for a generic point of the shell.

For a point on the shell middle surface, the covariant basis vectors defining the tangent plane to the middle surface are usually obtained as follows:

$$\vec{a}_\alpha = \vec{\Phi}(\xi^1, \xi^2)_{,\alpha}; \quad \vec{a}_3 = \frac{\vec{a}_1 \times \vec{a}_2}{||\vec{a}_1 \times \vec{a}_2||} \tag{2}$$

where $\vec{a}_3$ is the unit normal vector to the surface $\mathcal{S}$, see Fig. 1. In Eq. (2) and further on, latin indices $i, j, \ldots$ take their values in the set $\{1, 2, 3\}$ while greek indices $\alpha, \beta, \ldots$ take their values in the set $\{1, 2\}$. The summation convention on repeated indices is used and partial derivative is denoted by $()_{,\alpha}$.

A shell is characterized by the first fundamental form $a_{\alpha\beta}$ and the second one $b_{\alpha\beta}$. Their covariant, contravariant and mixed form definitions are given by:

$$a_{\alpha\beta} = \vec{a}_\alpha \cdot \vec{a}_\beta \quad a^{\alpha\beta} = \vec{a}^\alpha \cdot \vec{a}^\beta \quad b_{\alpha\beta} = \vec{a}_{\alpha\alpha,\beta} \cdot \vec{a}_3 \quad b_\alpha^\beta = \vec{a}^\beta \cdot \vec{a}_{3,\alpha} \tag{3}$$

For a generic point of the shell, covariant basis vectors must be defined and we have:

$$\vec{g}_\alpha = \vec{OM}(\xi, z)_{,\alpha} = (\delta_\alpha^\beta - z b_\alpha^\beta)\,\vec{a}_\beta = \mu_\alpha^\beta(z)\,\vec{a}_\beta \quad \text{and} \quad \vec{g}_3 = \vec{a}_3 \tag{4}$$

where δ_α^β is the Kronecker symbol and b_α^β is the mixed form of the second fundamental form. This basis $\vec{g}_i$, illustrated in Fig. 1, must be used to define quantities for any point of the shell. The form $\mu_\alpha^\beta(z)$ introduced in Eq. (4) defines the transport from the shell middle surface to any point of the shell and is associated with the curvature variation along the thickness direction z of the shell. The inverse tensor of the mixed tensor μ_α^β is denoted m_α^β and is defined as:

$$m_\alpha^\beta = (\mu^{-1})_\alpha^\beta = \frac{1}{\mu}\{\delta_\alpha^\beta + z\,(b_\alpha^\beta - 2H\delta_\alpha^\beta)\} \tag{5}$$