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Level set based topology optimization of vibrating structures for coupled acoustic–structural dynamics

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ABSTRACT

A level set based structural topology optimization method is proposed for the optimal design of coupled structural–acoustic system with a focus on interior noise reduction. The objective is to consider an optimum structure with an optimum interface between the structural and acoustic domains, for minimizing the acoustic response of the coupled system at specified points or surfaces inside the acoustic domain within a frequency range of interest, subject to the given amount of the material of the structure. A sensitivity analysis with respect to the structural boundary variations is carried out using material derivative and the adjoint method, while the standard finite element method is employed for solving the state equation and the adjoint equation. The optimal structure of the coupled structural–acoustic system with smooth boundary is obtained through the level set evolution, while the velocity field is derived from the sensitivity analysis and the optimization algorithm. A number of numerical examples are presented to demonstrate the feasibility and effectiveness of the proposed approach for the noise reduction purpose.

1. Introduction

Acoustic–structural optimization [1–5] is a method to improve certain acoustic characteristics of a structure. The process of acoustic–structural optimization is to establish an optimal structure with respect to its shape and topology and to result in better acoustic performance such as low noise level. Comparing to size optimization and shape optimization, topology optimization has been regarded as one of the most challenging and promising methods in structural optimization since the pioneering work of Bendsøe and Kikuchi [6]. And various families of topology optimization approaches, such as homogenization method [6], the solid isotropic material with penalization (SIMP) method [7], the evolutionary optimization method [8] and the level set method [9,10] have been developed.

In the past decades, topology optimization has been studied to solve a wide range of dynamic design problems for noise or vibration reduction, such as the fundamental eigenvalue maximization [11,12], the frequency response minimization [13–15], and the modal shape specification [16,17]. For the acoustic–structural optimization, the optimum material distribution (topology) of the structure has been the focus in the past. Luo and Gea [18] discussed the acoustic box topology optimization to obtain the optimal configuration of stiffeners for interior sound reduction. Lee and Wang [19] proposed to minimize the radiation and scattering of sound using the normal gradient integral equation and the genetic algorithms. Yoon et al. [20] presented a topology optimization method to minimize sound pressure levels within a prescribed acoustic cavity for a fully coupled structural–acoustic problem, using a mixed displacement/pressure finite element formulation. Du and Olhoff [21,22] proposed a SIMP based topology optimization method to minimize the sound power radiated from a structure surface placed inside an acoustic cavity, with sound pressure approximately evaluated by assuming plane waves. Lee et al. [23] proposed a one-dimensional topology optimization method to maximize the sound transmission loss, while the optimal layer sequencing and thickness of multilayered acoustical foam were found for several frequencies. Akl et al. [24] studied topology optimization of a plate coupled with an acoustic cavity to minimize the acoustic–structure interactions at different structural frequencies, while the plate thickness is chosen as the design variable.

However, the dynamics of the coupled acoustic–structural system depends also on the interface (shape) between the structural and acoustic domains. Therefore, an optimization of the interface (shape) is equally important as the structural topology optimization. Shape optimization of the structural–acoustic interface poses major difficulties for the density based optimization methods as there is no explicit representation of the interface in such methods,

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especially for smooth boundaries. Furthermore, solving the coupled acoustic–structural system state equation accurately becomes more challenging without an accurate model of the smooth interface geometry, because the primary variables of acoustic domain and structure domain interact at the interface through the boundary conditions on the interface [15,25].

In this paper we propose to employ a level set based topology optimization method [9,10] for the coupled acoustic–structural system. Under the level set scheme, the structural boundaries including the coupled interface are represented implicitly by the zero level set of an implicit function on a fixed Eulerian grid. The deformation of the structural boundaries including the coupled interface can be traced through the evolution of the level set function with respect to time. The evolution of the level set function with an advection velocity is performed by solving the Hamilton–Jacobi equation, where the advection velocity is derived from the sensitivity analysis and the optimization algorithm. During the whole optimization process, the complex shape and topology changes including the coupled interface can be simultaneously obtained. In addition, the direction and location of the design-dependent surface loads applied to the moving coupled interface can be specified in a straightforward way for the sensitivity analysis. Thus, the level set method is more amendable to the coupled acoustic–structural optimization problem than the density based topology optimization methods. To the best of our knowledge, this work represents an initial approach to the structural topology optimization of coupled acoustic–structural system using the level set method.

This paper is arranged as follows. In Section 2, a general formulation of structural optimization for coupled acoustic–structural system is given. In Section 3, the level set based structural topology optimization model is presented. In Section 4, the sensitivity analysis with respect to the structural boundaries is carried out, while the standard finite element method and the remeshing technique are employed to solve the state equation and the adjoint equation. In Section 5, the optimization algorithm and numerical scheme are described. In Section 6, several numerical examples of two-dimensional coupled acoustic–structural system are described to illustrate the proposed approach.

2. Coupled acoustic–structural system optimization problem

The coupled acoustic–structural system is shown in Fig. 1, with Ω_s and Ω_a denoting the structure and acoustic domains respectively. On the part of structure boundary Γ_σ and Γ_u, there are prescribed

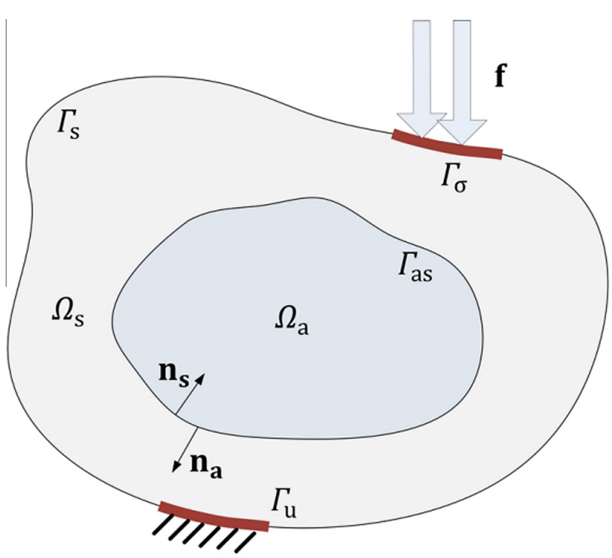

Fig. 1. Coupled acoustic–structural system.

boundary force $\mathbf{f}$ and displacement respectively. Γ_s is the outer boundary of the structure domain including Γ_σ and Γ_u. Γ_{as} is the coupled boundary between the structure and acoustic domains. The acoustic and structure domains move together in the normal direction of the boundary with the sound pressure p applied on the structure interface. We denote the opposite normal vectors of the structural boundary and the acoustic boundary by $\mathbf{n}^s$ and $\mathbf{n}^a$ respectively.

The state equations of the acoustic–structural interaction system, shown in Fig. 1, are given as [25],

$$\sigma_{ij,j}(\mathbf{u}) + \omega^2 \rho_s u_i = 0 \quad \text{in } \Omega_s \tag{1}$$

$$u_i = 0 \quad \text{on } \Gamma_u \tag{2}$$

$$\sigma_{ij}(\mathbf{u}) n_j^s - \mathbf{f}(\mathbf{x}) = 0 \quad \text{on } \Gamma_\sigma \tag{3}$$

$$\sigma_{ij}(\mathbf{u}) n_j^s - p n_i = 0 \quad \text{on } \Gamma_{as} \tag{4}$$

$$p_{,ii} + \frac{\omega^2}{c_a^2} p = 0 \quad \text{in } \Omega_a \tag{5}$$

$$p_{,i} n_i^a - \omega^2 \rho_a u_i n_i^a = 0 \quad \text{on } \Gamma_{as} \tag{6}$$

where $\mathbf{u}(\mathbf{x})$ is the displacement at point $\mathbf{x}$ ($\mathbf{x} \in \Omega_s$). $\mathbf{f}(\mathbf{x})$ is the dynamic load and $p(\mathbf{x})$ is the dynamic sound pressure at point $\mathbf{x}$ ($\mathbf{x} \in \Omega_a$). ρ_s and ρ_a are the structure mass density and the air mass density, respectively. The stress tensor $\sigma_{ij}(\mathbf{u}) = C_{ijkl}\varepsilon_{ij}(\mathbf{u})$ is related to the linearized strain tensor $\varepsilon_{ij}(\mathbf{u}) = (u_{i,j} + u_{j,i})/2$ and C_{ijkl} is the elasticity tensor. ω is the frequency of the harmonic dynamics.

Considering Eqs. (1)–(4), the weak form of the equilibrium equation for structure domain is obtained as follows:

$$\int_{\Omega_s} \sigma_{ij}(\mathbf{u})\varepsilon_{ij}(\mathbf{v}) d\Omega - \omega^2 \int_{\Omega_s} \rho_s u_i v_i d\Omega - \int_{\Gamma_\sigma} f_i v_i d\Gamma - \int_{\Gamma_{as}} p n_i v_i d\Gamma = 0 \tag{7}$$

Similarly, considering Eqs. (5) & (6), the weak form of the equilibrium equation for acoustic domain is given as follows:

$$\frac{1}{\rho_a} \int_{\Omega_a} p_{,i} q_{,i} d\Omega - \frac{\omega^2}{\rho_a c_a^2} \int_{\Omega_a} pq\, d\Omega - \omega^2 \int_{\Gamma_{as}} u_i n_i q\, d\Gamma = 0 \tag{8}$$

With the weak forms together, the state equation of coupled acoustic–structural system can be described as:

$$\begin{aligned} &k^s(\mathbf{u},\mathbf{v}) + k^a(p,q) - \omega^2 m^s(\mathbf{u},\mathbf{v}) - \omega^2 m^a(p,q) - c^s(p,\mathbf{v}) \\ &\quad - \omega^2 c^a(\mathbf{u},q) \\ &\quad = f(\mathbf{v}) \quad \forall q \in P,\ \forall \mathbf{v} \in U \end{aligned} \tag{9}$$

where $\mathbf{v}$ and q are the kinematically admissible virtual displacement and sound pressure, respectively, as U and P define their respective admissible space [25].

The following definitions are used to simplify notations for the integrands:

$$k^s(\mathbf{u},\mathbf{v}) = \int_{\Omega_s} C_{ijkl}\varepsilon_{kl}(\mathbf{u})\varepsilon_{ij}(\mathbf{v}) d\Omega \tag{10}$$

$$m^s(\mathbf{u},\mathbf{v}) = \int_{\Omega_s} \rho_s u_i v_i d\Omega \tag{11}$$

$$k^a(p,q) = \frac{1}{\rho_a} \int_{\Omega_a} p_{,i} q_{,i} d\Omega \tag{12}$$

$$m^a(p,q) = \frac{1}{\rho_a c_a^2} \int_{\Omega_a} pq d\Omega \tag{13}$$

$$c^s(p,\mathbf{v}) = \int_{\Gamma_{as}} p n_i v_i d\Gamma \tag{14}$$

$$c^a(\mathbf{u},q) = \int_{\Gamma_{as}} u_i n_i q d\Gamma \tag{15}$$

$$\mathbf{f}(\mathbf{v}) = \int_{\Gamma_\sigma} f_i v_i d\Gamma \tag{16}$$

The general problem of structure optimization for the coupled acoustic–structural system can be formulated as follows: