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Application of the pseudo-excitation method to assessment of walking variability on footbridge vibration

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ABSTRACT

The pseudo-excitation method is applied to determine the non-stationary vibration response of footbridges to variable pedestrian excitation. Excitation forces are described by their spectral densities, the models of which are taken from the literature. In addition a simple spectral model is proposed to encompass both intra- and inter-pedestrian variability. Comparison is made to Monte Carlo simulations of random pedestrian events and a means of estimating extreme statistics of random walking is given. The method is shown to be accurate and efficient. Consequently, this work should find value in explaining differences between observed and modelled responses.

1. Introduction

1.1. Background

Following well-known examples of footbridges that experienced excessive vibrations there has been increased research into the problem [1–3]. However, there remain differences between the predications of most loading models and measurements of actual responses. For accurate assessment of the serviceability limit state for footbridges, it is therefore essential that these differences be minimized, and that their sources are well understood.

The main source of difference between measured and modelled vibrations relates to the pedestrian and structure interaction, and the inherent variability of pedestrian walking [4–6]. Many measurements indicate both the obvious inter-pedestrian variability, and variability of the induced walking forces for the same pedestrian; termed intra-pedestrian variability. Recent models have been proposed to account for such variabilities [5–10]. Many of these models make use of a frequency spectrum of walking forces. However, a difficulty remains with the use of these spectra in that many simulations are required in a Monte Carlo framework to estimate the response of a footbridge to these force spectra and this can render their usefulness limited.

This work applies the pseudo-excitation method (PEM) to establish footbridge vibration response under pedestrian loading, considering imperfect walking, for both inter- and intra-pedestrian variability. This approach obviates the need for Monte Carlo simulations, allowing the response to be found with minimal computational effort for any force spectrum. Using this method, the main spectral models are evaluated and compared to a proposed simple spectral model that can account for both intra- and inter-pedestrian variability. While the general approach developed here is applied for the vertical loading case, it is readily applicable to the other important case of lateral vibration.

1.2. Pedestrian vertical loading

The vertical loads imparted by walking are often given by a Fourier series of the form [1,2]:

$$F(t) = W + \sum_{j=1}^{N} W\eta_j \sin(2\pi j f_p t - \varphi_j). \tag{1}$$

The terms η_j are the Fourier coefficients of the jth harmonic in the series, and therefore represent the dynamic loading factor (DLF) for that harmonic. Differing numbers of coefficients are used in the literature [10–14]. There is also great variability in the measured values, even for the same pedestrian [15,16]. Further, DLFs are found to vary with the frequency of the harmonic [17–20,9]. Table 1 gives some pertinent values.

1.3. Variability of pedestrian walking

The intra- and inter-pedestrian variability of walking forces is found by many authors. Ingólfsson [21] and Ingólfsson et al. [22]

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Nomenclature

The following symbols are used in this paper:

f	frequency variable, uniformly modulated evolutionary stochastic excitation
$\bar{f}$	normalized frequency parameter of Brownjohn et al. [5]
$\bar{f}_j$	frequency ratio of jth harmonic for the model of Zivanovic et al. [9]
$\tilde{f}$	pseudo evolutionary excitation
f_p	pacing frequency (Hz)
g	modulation function, acceleration due to gravity
k	time or distance step number
m	number of frequency divisions
m_P	pedestrian mass
$p_{\max}$	percentile of maximum response distribution
s_{XX}	one-sided auto-spectral density for single pedestrian walking
v_0	mean zero-crossing rate
x	input excitation process
$\tilde{x}$	pseudo excitation force, subscripts R and I indicate the real and imaginary components.
y	response process
y_{max}	maximum response
$\tilde{y}$	pseudo response, subscripts R and I indicate the real and imaginary components
A_i	parameter for ith harmonic of the simple spectral model
$A_{i,k}$	parameter for kth curve of ith harmonic in the models of Zivanovic et al. [9] and the empirical spectral model
$B_{i,k}$	parameter for kth curve of ith harmonic in the model of Zivanovic et al. [9] and the empirical spectral model
$C_{i,k}$	parameter for kth curve of ith harmonic in the model of Zivanovic et al. [9] and the empirical spectral model
$E[\cdot]$	expectation operator
F	evolutionary excitation random process
F_0	amplitude of sinusoidal excitation force
$F(t)$	pedestrian forcing function
$G(\cdot)$	Gumbel distribution function
$H(\cdot)$	frequency response function
$K(\cdot)$	kernel density function
M	number of points in discrete spectral density; number of time or distance steps
N	number of harmonics in pedestrian forcing function; number of frequency points, number of Monte Carlo simulations
S	one-sided auto-spectral density for excitation X, or response, Y, or modulated force F
T	duration of equivalent stationary process
$\mathrm{Var}[\cdot]$	variance operator
W	pedestrian weight
X	input random process
Y	response random process
α	Gumbel distribution location parameter
β	Gumbel distribution scale parameter
$\phi(\mu, \sigma)$	normal distribution probability density function with mean μ and standard deviation σ
φ_j	phase of the jth harmonic
γ	Euler constant, approximately 0.5772
η_j	dynamic load factor of the jth harmonic
λ_i	the ith spectral moment
$\mu_{s,i}$	mean of normal distribution of ith harmonic for single pedestrian in the simple spectral model
$\sigma_{s,i}$	standard deviation of normal distribution of ith harmonic for single pedestrian in the simple spectral model
$\sigma_{K,i}$	kernel density bandwidth for ith harmonic
Δf	frequency step

describe the variation of vertical force power spectral densities (PSDs) for both intra- and inter-pedestrian observed in a series of experiments. Ingólfsson and Georgakis [23] report on the variability of each of the first five harmonics of lateral force and present a method using Gaussian fits to the average of the PSDs around each harmonic. They also present the empirical distribution functions for each of the DLFs and use log-normal fits to model them. Ricciardelli and Pizzimenti [24] also report similar findings. Racic and Brownjohn [25] also provide results that show pedestrian vertical force shows considerable deviation from ideal Fourier series behavior. Brownjohn et al. [5] show that the assumption of perfect periodicity of pedestrian vertical force is not realized in practice by using a series of treadmill experiments. This work will be considered in detail further on. Finally, Li et al. [26] describe a novel approach using PSDs of pedestrian loading, based on the footfall forcing function and the arrival rates of pedestrians.

2. Spectral modeling of pedestrian-induced forces

2.1. Models in the literature

With the realization that the forces induced by walking are so variable both by the same person (intra-pedestrian) and between people (inter-pedestrian), the spectral approach to address the phenomenon is gaining increasing attention. There are many works that discuss this approach [6,27] but only a few that suggest spectral models for pedestrian-induced forces.

Brownjohn et al. [5] and Racic and Brownjohn [7] use treadmill experiments to show that perfect periodicity of pedestrian vertical force is not realized in practice. This 'imperfect' real walking is described by its auto-spectral density function and used to quantify the structural response to this real walking. Racic and Brownjohn [7] focus on lateral pedestrian forces and use a weighted sum of 17 Gaussian functions to fit the measured auto-spectral density.

Table 1
Some dynamic load factors (DLFs) from the literature.

Harmonic	Young [17]		ISO 10137 [18]		Brownjohn et al. [5]	
i	Frequency	DLF (η_i)	Frequency (Hz)	DLF (η_i)	Frequency (Hz)	DLF (η_i)
1	1–2.8	$0.41(f-0.95) \leqslant 0.56$	1.2–2.4	$0.37(f-1.0)$	1.3–2.4	$0.37(f-0.42)$
2	2–5.6	$0.069+0.0056f$	2.4–4.8	0.1	2.6–4.8	0.053
3	3–8.4	$0.033+0.0064f$	3.6–7.2	0.06	3.9–7.2	0.042
4	4–11.2	$0.013+0.0065f$	4.8–9.6	0.06	5.2–9.6	0.041
5	–	–	6.0–12.0	0.06	6.5–12.0	0.027
6					7.8–14.4	0.018

Note: for worst case, a phase angle of zero is used for all harmonics.