Computers and Structures 132 (2014) 55–64

Contents lists available at ScienceDirect

Computers and Structures

journal homepage: www.elsevier.com/locate/compstruc

Dynamic condensation approach to the calculation of eigensensitivity

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ARTICLE INFO

Article history:
Received 3 March 2013
Accepted 24 October 2013
Available online 5 December 2013

Keywords:
Dynamic condensation
Eigensensitivity
Eigenvalue derivative
Eigenvector derivative
Large structure

ABSTRACT

Calculation of the eigensensitivity of a large and complex structural system requires considerable computational resources and is time-consuming. This paper derives the eigenvalue and eigenvector derivatives of a structure based on a dynamic condensation technique. The eigensensitivity of a structure are computed by iteratively updating the derivatives of the condensed system matrices and a transformation matrix. As the condensed model is much smaller than the original full model, the proposed method is quite efficient in the calculation of the eigensensitivity. The accuracy and efficiency of the proposed method are verified by the GARTEUR structure and a cantilever plate.

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1. Introduction

An accurate finite element (FE) model is frequently required for the structural analysis in the aerospace, mechanical and civil engineering. To accurately describe the practical structure, the analytical model of a large-scale structure is usually represented by a complex model, including a large number of elements, degrees of freedom (DOFs) and structural parameters. The eigen-analysis based on the large-size model might take up considerable computation storage and be time-consuming. Model condensation (or reduction) method is an efficient technique to give fast computation of some lowest eigensolutions of the large structures [1].

Model condensation methods remove some DOFs (slave DOFs) of the original FE model and represent the discarded DOFs with the retained DOFs (master DOFs). Afterwards, the eigenfunction of the reduced model is solved to approximate the eigensolutions of the original structure [2–6]. Model condensation technique has been used in a variety of engineering and mechanical problems. Since the master DOFs have much smaller size than the full model, the computational resource and time are saved. In addition, the model condensation technique will be more promising if it is combined with the substructuring methods. Most substructuring methods constrain the interface coordinates of the substructures to recover the global structure [7–11]. If the interface DOFs are selected as the master DOFs to be analyzed, the substructuring method can be performed on the reduced model instead of the full model [9,10]. Finally, the model condensation technique has been used in the experimental modal analysis and related fields. In the experimental modal testing, the measured points are usually much fewer than the DOFs of the analytical model. The analytical models are required to be reduced to match the experimental counterparts. The model condensation technique is also useful in the determination of the sensor position in the experiments [12,13].

The model condensation methods have been widely developed to calculate the eigensolutions. Guyan [14] and Irons [15] firstly proposed the static condensation technique to calculate the eigensolutions, which neglected the inertia terms associated with the slave DOFs completely. It is exact at zero frequency and is acceptable for the lower frequency modes. O'Callahan [16] proposed the Improved Reduced System (IRS) method, which added an extra term to the static reduction transformation to include some inertia forces. Friswell et al. [17,18] developed a dynamic IRS strategy to achieve the accurate eigensolutions by an iterative scheme. Xia and Lin improved the dynamic IRS method, and proposed the iterative order reduction (IOR) method [2,3]. This improvement was proved to converge much faster than the dynamic IRS method, especially for the higher modes [2]. Qu et al. [5] defined the dynamic condensation matrix in the state space, and proposed an iterative dynamic condensation method for the model reduction of the viscously damped vibration systems. As the selection of master DOFs heavily influences the accuracy and efficiency of the model condensation methods, Jeong et al. proposed a rational

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http://dx.doi.org/10.1016/j.compstruc.2013.10.012

method for selecting the master DOFs based on the ratio of DOF-wise energy distributions [12].

The eigensensitivity is usually calculated together with the eigensolutions. It provides an estimate of the changes in the eigensolutions caused by the perturbations of the design parameters of a structural model. In model updating or optimization process, the eigensensitivity serves for indicating the searching direction of an optimization algorithm, which endows the more sensitive parameter (with respect to the objective function) a higher priority [9]. The eigensensitivity is usually calculated based on the full model of a structure [19,20]. Fox and Kapoor [21] firstly utilized the modal method to determine the eigenvalue and eigenvector derivatives with respect to the physical parameters in the mass and stiffness matrices. The modal method requires the superposition of the eigenmodes of the system to calculate the required eigenvalue and eigenvector derivatives. Nelson [22] proposed an exact method to calculate the eigenvector derivative of one mode by using the modal parameters of that mode only. Nelson's method has been further improved in terms of computational efficiency [23], and has also been generalized by taking into account the rigid body modes and/or the repeated modes [24,25]. Another approach for the eigensensitivity computation is based on an algebraic formulation [26]. The derivatives of each eigenvalue and its associated eigenvector are computed simultaneously by solving a group of algebraic equations. In the above methods, the eigensensitivity is calculated on the full model of a structure. As the measured points in the experiment are usually much fewer than the DOFs of the analytical model, the eigensensitivity of the analytical model is necessary to be reduced to match the experimental counterparts. It is time consuming and wasted to calculate the eigensensitivity of the full model and then reduce it to match the measured points.

This paper derives the eigensensitivity formula based on a reduced model using the dynamic condensation algorithm. The DOFs associated with the selected elemental parameter are retained as the master DOFs. Consequently, the change of the elemental parameter is localized within the stiffness and mass matrices of the master DOFs. The eigensensitivity of the reduced model can be obtained by directly performing iterations on the stiffness and mass matrices of the master DOFs, which consumes a small amount of computation time. The accuracy and efficiency of the proposed method are verified by the GARTEUR frame and a cantilever plate.

2. Basic dynamic condensation method for eigensolutions

In general, the free vibration of an undamped structure with N DOFs is described by the eigenequation [27]

$$(\mathbf{K}-\lambda_i\mathbf{M})\mathbf{\Phi}_i=\mathbf{0} \tag{1}$$

where $\mathbf{K}$ is the $N\times N$ symmetric stiffness matrix and $\mathbf{M}$ is the $N\times N$ symmetric mass matrix of the full model. λ_i is the ith eigenvalue, and $\mathbf{\Phi}_i$ is the associated mass-normalized eigenvector. If the full DOFs of a structure are divided into n_m master DOFs and n_s slave DOFs, the eigenequation is divided according to the master and slave DOFs into [2,3,16–18]

$$\left(\begin{bmatrix}\mathbf{K}_{mm} & \mathbf{K}_{ms}\\ \mathbf{K}_{sm} & \mathbf{K}_{ss}\end{bmatrix}-\lambda_i\begin{bmatrix}\mathbf{M}_{mm} & \mathbf{M}_{ms}\\ \mathbf{M}_{ms}^T & \mathbf{M}_{ss}\end{bmatrix}\right)\begin{Bmatrix}\mathbf{\Phi}_m\\ \mathbf{\Phi}_s\end{Bmatrix}_i=\begin{Bmatrix}\mathbf{0}\\ \mathbf{0}\end{Bmatrix} \tag{2}$$

The subscripts 'm' and 's' respectively represent the master and slave DOFs, and $N = n_m + n_s$. $\mathbf{\Phi}_m$ represents the eigenvector of the master DOFs, and $\mathbf{\Phi}_s$ represents the eigenvector of the slave DOFs. Superscript 'T' represents the transpose of a matrix. For convenience, λ and $\mathbf{\Phi}$ represent one mode only, and the subscript 'i' is omitted in the following analysis.

The second line of Eq. (2) gives

$$\mathbf{\Phi}_s=-(\mathbf{K}_{ss}-\lambda\mathbf{M}_{ss})^{-1}\left(\mathbf{K}_{ms}^T-\lambda\mathbf{M}_{ms}^T\right)\mathbf{\Phi}_m=\mathbf{t}\mathbf{\Phi}_m \tag{3}$$

where $\mathbf{t}$ is the transformation matrix in relating $\mathbf{\Phi}_m$ and $\mathbf{\Phi}_s$. In consequence, the complete eigenvector is represented by the master eigenvectors as

$$\mathbf{\Phi}=\begin{Bmatrix}\mathbf{\Phi}_m\\ \mathbf{\Phi}_s\end{Bmatrix}=\begin{Bmatrix}\mathbf{I}_m\\ \mathbf{t}\end{Bmatrix}\mathbf{\Phi}_m=\mathbf{T}\mathbf{\Phi}_m \tag{4a}$$

$$\mathbf{T}=\begin{bmatrix}\mathbf{I}_m\\ \mathbf{t}\end{bmatrix} \tag{4b}$$

where $\mathbf{T}$ is the transformation matrix in relating $\mathbf{\Phi}_m$ and $\mathbf{\Phi}$. In matrix $\mathbf{T}$, $\mathbf{I}_m$ is the unit matrix of order n_m, and $\mathbf{t}$ takes the form of [2,3]

$$\mathbf{t}=-(\mathbf{K}_{ss}-\lambda\mathbf{M}_{ss})^{-1}\left(\mathbf{K}_{ms}^T-\lambda\mathbf{M}_{ms}^T\right) \tag{5}$$

Substituting Eq. (4a) into Eq. (2) and pre-multiplying Eq. (2) by $\mathbf{T}^T$, one can obtain a reduced eigenequation of order n_m as [2,3]

$$(\mathbf{K}_R-\lambda\mathbf{M}_R)\mathbf{\Phi}_m=\mathbf{0} \tag{6}$$

where $\mathbf{K}_R = \mathbf{T}^T\mathbf{K}\mathbf{T}$ and $\mathbf{M}_R = \mathbf{T}^T\mathbf{M}\mathbf{T}$ are the reduced stiffness and mass matrices, and they can be written as

$$\begin{aligned}\mathbf{M}_R&=\mathbf{T}^T\mathbf{M}\mathbf{T}=\begin{bmatrix}\mathbf{I}_m & \mathbf{t}^T\end{bmatrix}\begin{bmatrix}\mathbf{M}_{mm} & \mathbf{M}_{ms}\\ \mathbf{M}_{sm} & \mathbf{M}_{ss}\end{bmatrix}\begin{bmatrix}\mathbf{I}_m\\ \mathbf{t}\end{bmatrix}\\ &=[\mathbf{M}_{mm}+\mathbf{M}_{ms}\mathbf{t}]+\mathbf{t}^T\left[\mathbf{M}_{ms}^T+\mathbf{M}_{ss}\mathbf{t}\right]\end{aligned} \tag{6a}$$

$$\begin{aligned}\mathbf{K}_R&=\mathbf{T}^T\mathbf{K}\mathbf{T}=\begin{bmatrix}\mathbf{I}_m & \mathbf{t}^T\end{bmatrix}\begin{bmatrix}\mathbf{K}_{mm} & \mathbf{K}_{ms}\\ \mathbf{K}_{sm} & \mathbf{K}_{ss}\end{bmatrix}\begin{bmatrix}\mathbf{I}_m\\ \mathbf{t}\end{bmatrix}\\ &=[\mathbf{K}_{mm}+\mathbf{K}_{ms}\mathbf{t}]+\mathbf{t}^T\left[\mathbf{K}_{ms}^T+\mathbf{K}_{ss}\mathbf{t}\right]\end{aligned} \tag{6b}$$

The reduced eigenequation (Eq. (6)) has the order of n_m, which is much smaller than the original eigenequation of order N. The system matrices $\mathbf{K}_R$ and $\mathbf{M}_R$ are frequency dependent and the eigen-problem cannot be solved directly by the usual eigensolver. Pre-multiplying Eq. (5) by the $(\mathbf{K}_{ss}-\lambda\mathbf{M}_{ss})$, one has

$$(\mathbf{K}_{ss}-\lambda\mathbf{M}_{ss})\mathbf{t}=-\left(\mathbf{K}_{ms}^T-\lambda\mathbf{M}_{ms}^T\right) \tag{7}$$

From Eq. (7), the transformation matrix $\mathbf{t}$ can be written as

$$\mathbf{t}=-\mathbf{K}_{ss}^{-1}\mathbf{K}_{ms}^T+\lambda\mathbf{K}_{ss}^{-1}\left(\mathbf{M}_{ms}^T+\mathbf{M}_{ss}\mathbf{t}\right)=\mathbf{t}_G+\mathbf{t}_d \tag{8}$$

where

$$\mathbf{t}_G=-\mathbf{K}_{ss}^{-1}\mathbf{K}_{ms}^T \tag{8a}$$

$$\mathbf{t}_d=\lambda\mathbf{K}_{ss}^{-1}\left(\mathbf{M}_{ms}^T+\mathbf{M}_{ss}\mathbf{t}\right)=\mathbf{K}_{ss}^{-1}\left(\mathbf{M}_{ms}^T+\mathbf{M}_{ss}\mathbf{t}_G+\mathbf{M}_{ss}\mathbf{t}_d\right) \tag{8b}$$

Subscript 'G' represents a item of the Guyan static condensation [14], which is a static item and is determined directly without iteration. Subscript 'd' represents a dynamic item, which is frequency dependent and will be achieved by an iteration process.

The dynamic stiffness matrix of the reduced model can be rewritten as [2,3]

$$\begin{aligned}(\mathbf{K}_R-\lambda\mathbf{M}_R)&=[\mathbf{K}_{mm}+\mathbf{K}_{ms}(\mathbf{t}_G+\mathbf{t}_d)]+(\mathbf{t}_G+\mathbf{t}_d)^T\left[\mathbf{K}_{ms}^T+\mathbf{K}_{ss}(\mathbf{t}_G+\mathbf{t}_d)\right]\\ &\quad-\lambda[\mathbf{M}_{mm}+\mathbf{M}_{ms}(\mathbf{t}_G+\mathbf{t}_d)]+(\mathbf{t}_G+\mathbf{t}_d)^T\left[\mathbf{M}_{ms}^T+\mathbf{M}_{ss}(\mathbf{t}_G+\mathbf{t}_d)\right]\\ &=\mathbf{K}_G-\lambda\mathbf{M}_d\end{aligned} \tag{9}$$

where

$$\mathbf{K}_G=\mathbf{K}_{mm}-\mathbf{K}_{ms}\mathbf{K}_{ss}^{-1}\mathbf{K}_{ms}^T \tag{9a}$$

$$\mathbf{M}_d=[\mathbf{M}_{mm}+\mathbf{M}_{ms}\mathbf{t}]+\mathbf{t}_G^T\left[\mathbf{M}_{ms}^T+\mathbf{M}_{ss}\mathbf{t}\right] \tag{9b}$$

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