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Form-finding of compressive structures using Prescriptive Dynamic Relaxation

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ABSTRACT

This paper presents an adaptation of the Dynamic Relaxation method for the form-finding of small-strain compressive structures that can be used to achieve project-specific requirements such as prescribed element lengths. Novel truss and triangle elements are developed to permit large strains in the form-finding model while anticipating the small-strain behavior of the realized structure. Forcing functions are formulated to permit element length prescription using a new iterative technique termed Prescriptive Dynamic Relaxation (PDR). Case studies of a segmental concrete shell and a pedestrian steel bridge illustrate the potential for using PDR to achieve economic and environmentally considerate structural solutions.

1. Dynamic Relaxation and structural form-finding

Dynamic Relaxation (DR) was first proposed by Day in 1965 as an alternative analysis tool for indeterminate structures [1]. Using equations derived from the second law of motion, DR transforms a nonlinear static problem into a pseudo-dynamic one in which the displacements are updated via a time-stepping procedure to achieve a sufficiently equilibrated state. Since its inception, DR has been used as a nonlinear solver for a broad range of analytical problems [2] but was first used as a form-finding tool for tensile structures by Barnes [3–6]. DR has since been employed for the form-finding of cable-membrane structures [7], grid shells [8,9], continuous shells [10,11], and tensegrity structures [12,13].

Form-finding techniques can be assigned to three categories: physical hanging models, equilibrium methods, and optimization schemes. Physical hanging models, like those used by Antoni Gaudi [14], Heinz Isler [15], and Frei Otto [16], typically rely on inextensible cable networks to create purely axial systems under a gravitational load. Equilibrium methods such as Dynamic Relaxation, force density [17], stress distribution [18], thrust-network analysis [19], and particle-spring [20] use iteration algorithms to manipulate nodal geometry to equilibrate method-specific internal forces with applied external loads. Optimization schemes manipulate control parameters, such as nodal coordinates, of a structural system to provide an optimal solution for one criterion [21] or provide a Pareto Front for multiple criteria [22].

An example of a simple form-finding problem is the two-element truss shown in Fig. 1a. The basic formulation for form-finding is to determine coordinates for unconstrained nodes such that the system is in equilibrium. In this case, equilibrium is an insufficient constraint for the form-finding process to be useful; equilibrium would only restrain the free node from being positioned on the horizontal axis of the supports. Additional requirements can be introduced, for instance that all elements are in compression (Fig. 1b); that both elements are equally loaded (Fig. 1c); or that the right element is a certain length (Fig. 1d). A union of these requirements would produce an intersection of solution spaces resulting in one solution (Fig. 1e) or no solution at all (Fig. 1f).

If the form-finding model is based on an equilibrium approach, then the form-found shape will be influenced by the internal forces experienced by the elements of the form-finding model. Depending on the structural system, the forces in the model may differ from those generated in the elements of the realized static structure. For a determinate structure (Fig. 1), there exists only one solution for the internal forces in the structure thereby requiring that the element forces of the form-finding model match those of the realized static structure. For an indeterminate structure, such as the one shown in Fig. 2a, the distribution of forces will depend not only on the form-found shape, but also on the relative stiffnesses of the elements in the realized static structure. If the stiffness of any one of the elements is negligible compared to the others, that element will take negligible load (Fig. 2b–d). A desirable asset for a form-finding technique is to be able to anticipate the stiffness of the static structure. While geometry supplies one of the components of

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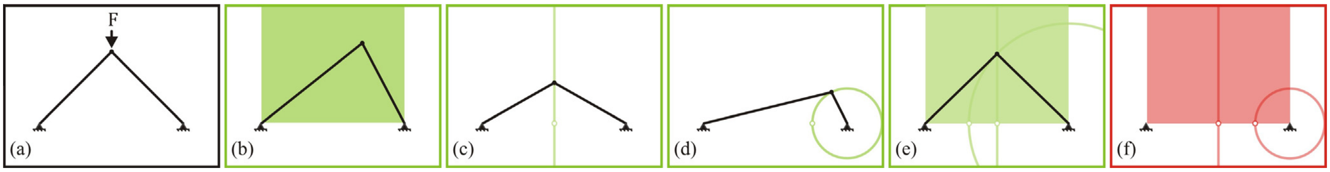

Fig. 1. Form-finding of a determinate structure. Shaded areas indicate the search space imposed by the constraints.

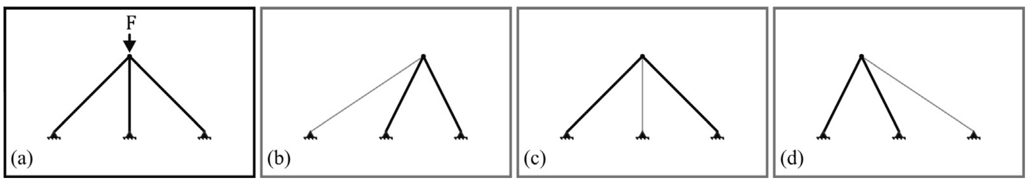

Fig. 2. Form-finding of an indeterminate structure.

static stiffness, material properties and element dimensions also contribute.

Neither equilibrium methods nor physical models have typically afforded significant opportunity for introducing material properties; in fact, a recent review of form-finding techniques identified the form-finding process as 'material independent' [23]. It is also common to consider the form-found shape as a starting point to which dimensions can be assigned [24], e.g., by density distribution [25,26]. While optimization schemes rely on computational models with accurately defined material properties, they are not well suited for finding funicular shapes. For compressive systems, the most efficient form is one that relies on a resolution of external loads through axial internal forces [27]. The physical hanging models exemplify Hooke's frequently used adage, 'as hangs the flexible cable so, inverted, stand the touching pieces of an arch,' [28] by relying on cables that cannot resist bending to produce shapes that when inverted are entirely in compression.

It is possible to identify three desirable qualities for a form-finding process for compressive systems:

1. Elements can only transmit axial loads
2. Material properties and dimensions of the realized structure are included as parameters in the form-finding process
3. Project-specific requirements can be introduced systematically

Because DR is rooted in the analysis of real structures, it is well suited for the form-finding of cable-membrane structures as it simulates realistic structural behavior [7]. Accordingly, the authors propose that DR is the best suited of the equilibrium methods for incorporating realistic material properties to produce an axially-driven form-finding process for compressive structures. The basic DR algorithm used for this study is presented in Section 2. In Section 3, we introduce a truss element and a triangular membrane element for the form-finding of compressive structures. In Section 4, we introduce the concept of Prescriptive Dynamic Relaxation (PDR), which permits the achievement of certain system requirements through a modification of the DR process. In Section 5, we offer a method to achieve prescribed element lengths using forcing functions in PDR. In Section 6, we provide case studies demonstrating application to a concrete shell and a steel pedestrian bridge.

2. The Dynamic Relaxation algorithm

The DR method presented in this section is adapted from Barnes [5]. First, the Residual, $R^t_{i,x}$, at time t is calculated:

$$R^t_{i,x} = P_{i,x} + \sum_j \sum_{co(k)=i} F^t_{i,j,k,x} \quad (1)$$

where the indices i, j, k, and x refer to global node number, element number, local node number, and directional degree of freedom; the $co()$ operator converts local numbering to global numbering; $P_{i,x}$ is the applied external load; and $F^t_{i,j,k,x}$ is the element force vector. Next, the updated velocity, $V^{t+\Delta t/2}_{i,x}$, is found:

$$V^{t+\Delta t/2}_{i,x} = \begin{Bmatrix} V^{t+\Delta t/2}_{i,x} + \frac{\Delta t}{M_i} R^t_{i,x} & \text{if} & c_{i,x} = 0 \\ 0 & \text{if} & c_{i,x} = 1 \end{Bmatrix} \quad (2)$$

where Δt is the time step, M_i is the fictitious nodal mass, and $c_{i,x}$ is the binary restraint value for the degree of freedom (0 if free, 1 if restrained). The new nodal coordinates, $x^{t+\Delta t}_i$, are then found:

$$x^{t+\Delta t}_i = x^t_i + \Delta t V^{t+\Delta t/2}_{i,x} \quad (3)$$

To reach equilibrium, it is necessary to damp the system. Day introduced viscous damping by multiplying the velocity term, $V^{t-\Delta t/2}_{i,x}$, by an arbitrary damping constant, $0 < K_V < 1$ [1]. Kinetic damping, an alternative to viscous damping, was first introduced by Cundall in 1976 [29]. To achieve kinetic damping, the kinetic energy, K^t, is tracked at each iteration:

$$K^t = \sum_i M_i |V^{t-\Delta t/2}_i|^2 \quad (4)$$

When the kinetic energy is at a maximum (corresponding to minimum strain energy), the velocity is set to zero. The iterations are terminated when the system achieves a prescribed level of equilibrium. In this paper, a stringent criterion, $f_{conv} \ll 1$, is implemented:

$$\max_{\forall i} \sqrt{\frac{\sum_x \left(R^t_{i,x}\right)^2 (1 - c_{i,x})}{\sum_x (P_{i,x})^2 + (W^o_i)^2}} \leq f_{conv} \quad (5)$$

The numerator is the maximum of the current residuals, $R^t_{i,x}$, and the denominator is the maximum of the applied loads calculated as a sum of the external loads, $P_{i,x}$, and initial self-weight element contributions to each node, W^o_i.

The DR iterative process can be summarized in three basic steps:

1. Initialize model (e.g., starting geometry, material properties, boundary conditions, and loading)
2. Calculate element forces and residuals. If f_{conv} is achieved, output results and terminate.
3. Calculate velocities (adjusted to chosen method of damping) and nodal coordinates. Go to step 2.