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Development of a novel phase-field method for local stress-based shape and topology optimization

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ABSTRACT

This research develops a stress-based topology optimization method (STOM) using the phase-field method representing topological changes. This research shows that to apply the phase field method, regional and localized stress constraints should be addressed. Thus, we use an Augmented Lagrange multiplier approach for the stress constraints and present a new numerical solution for the Lagrange multipliers inside the Allen–Cahn equation with the topological derivatives. Through several two dimensional illustrative problems, the results of the phase-field method have larger objective values, but are robust from a stress point of view compared with the results of the STOM by the density method.

1. Introduction

Limiting the maximum nominal stress of linear and nonlinear structures has become an important engineering problem [1–10]. When nominal stress (either due to a static or dynamic load) exceeds a certain limit, static fracture or dynamic fatigue failure leading to catastrophic disasters occurs. To prevent these failures, a practical engineering approach is to calculate the nominal stress values of a structure of interest by finite element method and to confine them to a certain maximum value by changing the geometry or the material of a structure of interest. Furthermore, size and shape optimizations for stress constraints have been researched for a long time, and these results are commonly applied to obtain safer and more robust designs from mathematical and engineering points of view. The use of topology optimization (TO) methods to consider local stress constraints defined at every finite element is a recent achievement. This research contributes to these optimization researches by presenting a new stress-based shape and stress-based topology optimization method (STOM) using the phase-field method that expresses topological changes in a design domain with explicit phase-field curves. Despite some relevant works on STOM with the density design variables or the level set function variables [1–4,6–13], optimization methods minimizing volume subject to stress constraints defined at all finite elements (hereafter local stress constraints) have not yet been proposed using the phase-field method due to several difficulties.

1.1. Issues for local stress constraints

In conducting the stress-based topology optimization, the three well-known difficulties, i.e., the singularity problem, the local constraint problem, and the highly nonlinear behavior of stress constraints, should be properly addressed [1–4,6–10,14–17]. The singularity behaviors of the stress constraints in TO have been addressed [4,6,14,15,17]. According to the previous researches, stress singularities arise when some design variables of the SIMP method converge to the lower bound (i.e., 0.001 or 0.0001) to simulate nonstructural regions (“void” regions). To resolve this issue, there have been many proposed solutions and relaxation methods such as the *epsilon* relaxation method [4,17], the *qp*-relaxation method [1,2], and the relaxed stress indicator method [6]. Second, as the nominal stress values of all finite elements of interest must be constrained, from a computational point of view there are too many constraints to efficiently solve the optimization problem with a dual optimizer. As the computational cost for the sensitivity analysis and the sub-optimization increases, one must resort to approximation methods and other remedies. One of the methods is the constraint selection method, which selects only active stress constraints and calculates their sensitivity values [4]. Recently, the global stress measure methods have been proposed [6,9,18]. Until now, the popular two proposals are the *p*-norm approach and the Kreisselmeier–Steinhauser (KS) approach. In this paper, the *p*-norm approach is employed with a correction or scaling factor (see [6,8,9,18] for more detail). In addition, with only one global constraint measure, it is impossible to consider the effects of the localized stress constraints accurately. Thus a method of dividing the design domain into several sub-

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regions and calculating the global stress measures at each domain is employed in this research [6,8,9]. For the third issue, it is important to appropriately consider the highly nonlinear behavior of the stress constraints due to the relaxation, the penalization of the design variables of TO [19], and the global stress measure. To resolve this issue, an efficient and accurate dual sequential approximate optimization (SAO) method should be employed. We will discuss each of these issues as they relate to our phase-field method approach.

1.2. Post-processing issues

The post-processing problem is also important in the stress-based topology optimization problem. From a mathematical point of view, the original TO problem of finding the so called "solid" and "void" domains inside a design domain is a binary optimization problem that is almost impossible to apply to practical engineering problems. Therefore, it has been common to *relax* the problem by introducing the continuous design variables of the homogenization method or using the Solid Isotropic Material with Penalization (SIMP) method. With such a relaxation, it becomes possible to obtain optimal topological layouts within a reasonable computation time, but a post-processing of the final layouts with the intermediate design variables of the relaxed TO problem should be completed. Fig. 1 shows a crude post-processing result of an *L*-shaped beam structure using the SIMP method and the hard-kill post-processing which sets the design variables to 1 or 0 depending on a threshold value, which is 0.5 in Fig. 1. As shown in Fig. 1(b), the maximum stress value may be severely increased after the postprocessing. Therefore, to represent layouts more precisely and explicitly during and after TO, some applications of the level set method [20–22] or the phase-field method [23] have been proposed. As some explicit curves between the solid and void domains are parameterized and optimized, the intermediate design variables are less presented, and the post-processing becomes relatively straightforward. Thus, this research addresses intermediate design variables and post-processing with the phase-field method, and investigates the differences between the density-based method and the phase-field method from a structural point of view.

1.3. Augmented Lagrange multipliers for multiple stress constraints

To the best of our knowledge, the previous works with the phase field method has considered the case of only one constraint, such as a volume constraint. Usually, a structural performance measure such as compliance, target displacement or the weighted sum of squares of the eigenfrequencies is chosen [22–24]. However, the stress-based TO with the *p*-norm stress measures in multiple sub-regions, the mathematical consideration of multiple constraints becomes an important issue [6,9,11,18]. To resolve this issue in the phase field method, this research presents an Augmented Lagrange multiplier (ALM) method to transform an optimization problem with multiple constraints into an optimization problem with a single objective function without constraints by employing Lagrange multipliers.

1.4. Design space comparison

As another issue, the differences in the size of the design spaces between the phase-field method and the SIMP method should be considered. Compared with the design space of the SIMP method, the design space of the phase-field method is smaller. To enlarge the limited design space of the phase-field method, heuristic topological derivative methods have been developed [25,26]. Although the mathematical formulations of the topological derivative methods are highly sophisticated, their implementation with finite element methods becomes heuristic. Furthermore, it is likely that despite the introduction of topological derivative methods, it is still difficult to obtain better designs than the SIMP method. Thus, it is natural for us to study the differences in the design spaces of the SIMP method and the phase-field method. Our concern regards the effect of the smaller design space of the phase-field method on the optimal layout in the stress based topology optimization. In particular, we investigated the following two questions. Firstly, the differences of highly stressed regions in the phase-field method and the SIMP approach are investigated. From the previous discussions [6,11], it seems that smooth curves and features can be obtained in the SIMP method to prevent stress concentration, and the question is whether the same is true of the phase-field method. Secondly, we analyze the roles of the internal members of the optimized layouts. In other words, we want to investigate whether internal members appear to minimize compliance or confine stress values. Furthermore, we also derived and implemented a new formulation for the regional stress constraints of the topological derivative method. The phase-field method with an explicit curve function has a smaller design space compared with SIMP-based topology optimization. For this reason, as iterations proceed, there is a tendency in the phase-field method to erase void regions, which in turn limits the design space. To resolve this issue, we propose a topological derivative method that heuristically introduces some holes. We calculated and implemented the topological derivatives of the *p*-norm stress constraint functions per every fixed number of evolution cycles. The detailed procedure will be presented in Section 3.

The remainder of the paper is organized as follows. In Section 2, we give a brief introduction to the phase-field method and its application to general TO problems. Section 3 gives the formulation of the stress-based TO problem we consider and a detailed description of the proposed optimization procedure using the ALM and topological derivatives via the phase-field method. The usefulness of the present method will be verified by solving several structural optimization problems in Section 4. Section 4 also compares the phase-field method with the SIMP method. Finally, the conclusion summarizes our findings.

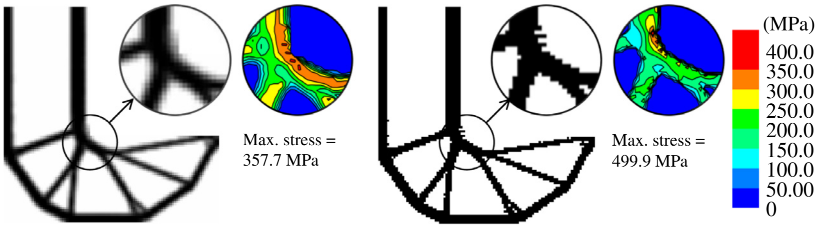

Fig. 1. Crude post-processing of the intermediate design variables from a stress point of view.