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The solution of Maxwell's equations in multiphysics

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ABSTRACT

We consider the solution of the fully-coupled equations of electromagnetics with fluid flows and structures. The electromagnetic effects are governed by the general Maxwell's equations, the fluid flows by the Navier–Stokes equations, and the solids and structures by the general Cauchy equations of motion. We present an effective general finite element formulation for the solution of the Maxwell's equations and demonstrate the coupling to the equations for fluids and structures. For the solution, we can use the electric field and magnetic field intensities, or the electric and magnetic potentials, with advantages depending on the problem solved. We give various example solutions that illustrate the use of the solution procedure.

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1. Introduction

During the recent years, an increasing emphasis has been placed on the solution of multiphysics problems [1]. While the solution of problems considered separately in solids and structures, in fluid flows, and in electromagnetics (EM) has been pursued for decades – and widely-used quite powerful computer programs are now available – the solution of problems in which general structures interact with fluid flows and electromagnetic waves has hardly been tackled and presents special difficulties. Indeed, only specific problems have been solved in which the solution techniques have been developed specifically for the physical problem considered, see for example, refs. [2–11].

Considering the analysis of solids and structures coupled with fluid flows, many publications have recently appeared, and numerous applications are found, in particular, in biomechanics and the automotive and airplane industries. The next step for general multiphysics solutions is clearly that electromagnetic effects should also be included. In today's time, electrical devices are used daily by almost everybody in a multitude of applications, and to reach optimal designs the structural, fluid and electromagnetic fully-coupled effects would ideally be considered. These coupled effects can be particular important, for example, in problems of magneto-solid and fluid mechanics, in medical applications and biomedical engineering, metal processing, and plasma physics, see ref. [12] and the references therein.

Numerous publications are also available on the numerical solution of electromagnetic field problems. In the most general cases, the general Maxwell's equations are considered. However, while finite element solutions have been obtained for some decades, the earliest attempts frequently showed spurious modes and in that sense were not reliable [13,14]. Thereafter, special finite element schemes were designed, and in particular the edge-based elements [15]. These elements are more reliable but have the shortcomings that the edge degrees of freedom are difficult to couple with the usual nodal degrees of freedom used in the finite element analyses of fluid flows and structures, divergence-free conditions are considered, the convergence is not optimal, and the elements do not directly fit into the usual post-processing schemes used. In more recent research, various discontinuous finite element schemes and meshless methods have been proposed, see for example, Nicomedes et al. [16] and Badia and Codina [17], but these procedures are computationally quite costly or contain artificial numerical factors for general practical analyses.

Our objective in this paper is to present a novel finite element scheme for the solution of the general Maxwell's equations specifically developed to solve for electromagnetic effects coupled with fluid flows, solids and structures, while keeping our philosophy for the development of finite element procedures in mind [18]. Since we amply published on our solution procedures for fluid flows with structural interactions previously [19–22], we focus in this paper on the solution of Maxwell's equations, to couple with the governing equations of fluids and structures. We consider the static and harmonic solutions of the Maxwell's equations, including the solution of high-frequency problems, and present a general uniform procedure for solution in which either the primitive

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variables of electric and magnetic fields are used (**E** and **H**), or the scalar electric and vector magnetic potentials (ϕ and **A**) are employed. The finite elements used are similar to those we proposed for the solution of the Navier–Stokes equations, and the full coupling between the different physical phenomena is achieved as in the schemes for fluid-structure interaction analysis [20–22].

In the next sections, we first review the general Maxwell's equations and the form in which we use these equations for our discretization scheme. We pursue static and harmonic solutions, and give some attention to the boundary conditions that arise in practice. Since we consider the solution using primitive variables and potentials, and two- and three-dimensional analyses, the use of the appropriate formulation can be important. Finally, we present the results of some fluid and structural problems with electromagnetic effects that illustrate the procedures proposed in this paper.

2. The electromagnetic governing equations

In this section we first summarize the original first-order Maxwell's equations, and then focus on the reformulation of the equations to the form in which we are solving them using a novel finite element scheme for electromagnetics (but previously published for fluid flows [19]). In a typical analysis, we consider the electric, magnetic, fluid and structural domains, possibly with heat transfer.

The computational domain can be two- or three-dimensional, and in total may consist of an electric domain Ω_e and/or magnetic domain Ω_m. In general, the coupled analysis also includes the structural domain, Ω_s, and the fluid domain, Ω_f. These domains may be partially or fully coincident, as illustrated in Fig. 1.

In the following we focus on the electromagnetic effects, but also discuss how these effects are coupled into the structural and fluid flow phenomena. For the solution of the fluid flows and structural responses, the discretizations used and the coupling procedures, we refer to refs. [19–23].

2.1. Original Maxwell's equations

The original first-order full Maxwell's equations [24], in general static or time-varying fields, can be written as Faraday's law

$$\nabla \times \mathbf{E} = -\mathbf{K} \tag{1}$$

and Ampère's law with the Maxwell term

$$\nabla \times \mathbf{H} = \mathbf{J} \tag{2}$$

where **E** and **H** are the electric and magnetic field intensities, and **J** and **K** are electric and magnetic current density source terms given below.

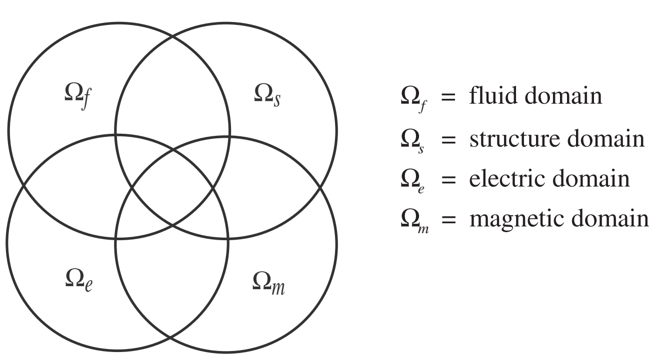

Fig. 1. Schematic diagram of physical domains that can be considered; the domain disks can be moved and any one of the domains may not be present.

The additional equations are the Gauss law applied to the electric and magnetic fields

$$\nabla \cdot \mathbf{D} = \rho_0 \tag{3}$$

$$\nabla \cdot \mathbf{B} = 0 \tag{4}$$

where

$$\mathbf{D} = \varepsilon \mathbf{E}, \quad \mathbf{B} = \mu \mathbf{H} \tag{5}$$

and ε, μ are the permittivity and permeability, respectively, of the material in the fields, and ρ_0 is a charge density source (the value being dependent on the equation).

Here we have

$$\begin{aligned} \mathbf{J} &= \mathbf{J}_0 + \sigma \mathbf{E} + \frac{\partial \mathbf{D}}{\partial t} \\ \mathbf{K} &= \mathbf{K}_0 + \frac{\partial \mathbf{B}}{\partial t} \end{aligned} \tag{6}$$

where $\mathbf{J}_0$ and $\mathbf{K}_0$ are the imposed electric and magnetic current densities respectively, and σ is the electric conductivity of the medium.

In the above equations, we do not point out the domains in which the variables are computed, because the context itself will imply where the variables are applicable. For the same reason, we also do not specify the individual computational domains in the following.

We consider the harmonic and static cases. In a harmonic analysis, all variables are expressed as $\mathrm{Re}(f^* e^{i\omega t})$ with a prescribed angular frequency ω, $f^* = f_r + if_i$ where i is the imaginary unit, and we have in phasor form

$$\begin{aligned} \mathbf{J} &= \mathbf{J}_0 + i\omega\varepsilon^* \mathbf{E} \\ \mathbf{K} &= \mathbf{K}_0 + i\omega \mathbf{B} \end{aligned}$$

where $\varepsilon^* = \varepsilon - i\sigma/\omega$.

In static analysis, naturally, all time-dependent terms vanish and all imaginary components are not present.

We should also note that, once the complex response has been calculated, the actual solution is given by

$$f = f_r \cos \omega t - f_i \sin \omega t$$

In our numerical solutions we solve for f_r and f_i using real arithmetic.

2.2. *E–H* formulation of Maxwell's equations for finite element solution

The second-order equation system is obtained by applying the operator $\nabla\times$ to Eqs. (1) and (2) to obtain

$$\nabla \times \nabla \times \mathbf{E} = -\nabla \times \mathbf{K} \tag{7}$$

$$\nabla \times \nabla \times \mathbf{H} = \nabla \times \mathbf{J} \tag{8}$$

Introducing as additional solution variables

$$p = \nabla \cdot \mathbf{E} - \rho_0/\varepsilon^* \tag{9}$$

$$q = \nabla \cdot \mathbf{H} \tag{10}$$

Eqs. (7) and (8) can be written as

$$\nabla \cdot ((p + \rho_0/\varepsilon^*)\mathbf{I} - \nabla \mathbf{E} + \mathbf{I} \times \mathbf{K}) = \mathbf{0} \tag{11}$$

and

$$\nabla \cdot (q\mathbf{I} - \nabla \mathbf{H} - \mathbf{I} \times \mathbf{J}) = \mathbf{0} \tag{12}$$

where **I** is the identity tensor.

Eqs. (11), (12), (3), and (4) form the **E**–**H** mathematical formulation that we use for our finite element solution. This formulation is specialized to specific cases, when appropriate, by omitting certain equations. We note that we use the divergence form of the

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