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A locking-free stabilized kinematic EFG model for plane strain limit analysis

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ABSTRACT

An element free Galerkin (EFG) based formulation for limit analysis of rigid-perfectly plastic plane strain problems is presented. In the paper it is demonstrated that volumetric locking and instability problems can be avoided by using a stabilized conforming nodal integration scheme. Furthermore, the stabilized EFG method described allows stable and accurate solutions to be obtained with minimal computational effort. The discrete kinematic formulation is cast in the form of a second-order cone problem, allowing efficient interior-point solvers to be used to obtain solutions. Finally, the performance of the stabilized EFG method is illustrated by considering several numerical examples.

1. Introduction

The load required to cause collapse of a body or structure can be directly estimated using limit analysis. Considering an upper-bound kinematic limit analysis approach, the flow rule is required to be satisfied everywhere in the problem domain. This requirement can easily be met using constant strain finite elements. However, it is well-known that such elements exhibit volumetric locking phenomena when used in conjunction with von Mises or Tresca yield criteria, due to the large number of incompressibility constraints imposed on the nodal velocities [1–3]. Various solutions have been proposed in the literature to overcome this problem. These include the use of higher-order displacement-based finite elements [3,4], mixed formulations [2,5–7] and kinematic formulations using discontinuous velocity fields [8–10]. Additionally a fully discontinuous formulation which involves identification of the critical layout of discontinuities at failure has been proposed [11].

Recently, Le et al. [12] proposed a numerical kinematic formulation using the cell-based smoothed finite element method (SFEM) and second-order cone programming (SOCP) to prevent the volumetric locking problem, and also to furnish good (approximate) upper-bound solutions for plane strain problems governed by the von Mises failure criterion. Alternately, meshfree methods can be used. The element-free Galerkin (EFG) method, one of the first meshfree approaches, has been applied successfully to a wide range of computational problems, proving popular due to its naturally conforming property (with no nodal connectivity required) and its rapid convergence characteristics [13]. The EFG method has also been applied successfully to limit analysis problems [14–16]. It has been shown that the EFG method is in general well suited for limit analysis problems, allowing accurate solutions to be obtained with relatively few degrees of freedom. Following this line of research, the main objective of this paper is to investigate the performance of a stabilized EFG method when applied to plane strain limit analysis problems, where volumetric locking can occur as a result of the use of an unbounded yield criterion.

Volumetric (or 'isochoric') locking is caused by the use of approximations which prevent certain velocity fields from being exactly described [17]. When low-order finite elements are used, the kinematic constraint (or 'divergence-free' or 'incompressibility' condition) leads to a reduction in the available number of degrees of freedom, and therefore the true velocity field cannot be exactly described. However, meshfree methods generally provide high -order shape functions [13,18], and therefore volumetric locking in elasto-plastic analysis problems can be suppressed by increasing the so-called dilation parameter [17,19,20], though not entirely removed [21]. The locking problem can also be relieved by using direct nodal integration or collocation methods, but these methods often result in rank deficiency and thus can produce spurious singular modes [22,23]. In order to eliminate the spatial instabilities associated with nodal integration, a stabilized conforming nodal integration (SCNI) has been proposed in [24], which has then been applied successfully to various problems [16,23,25,26]; see also [27] for a description of how kinematic and equilibrium approaches can be used in combination to obtain close bounds on the exact solution for plate problems. In the present paper, which

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focuses on plane strain problems, it will be shown that when the SCNI scheme is employed in the EFG-based kinematic formulation, the solutions obtained are accurate and stable, and volumetric locking can also be prevented.

This paper is organized as follows: in the next section, the kinematic limit analysis formulation is briefly reviewed. The approximation used to describe the displacement field and the SCNI smoothing technique are then presented, and the discrete formulation is also given. In Section 4, the underlying optimization problem is cast in the form of a second-order cone problem, allowing efficient interior-point solvers to be used to obtain solutions. Numerical examples are provided in Section 5 to illustrate the ability of the proposed method to prevent volumetric locking, and approximated upper bound solutions are then compared with those in the literature.

2. Kinematic limit analysis

Consider a rigid-perfectly plastic body of area $\Omega \in \mathbb{R}^2$ with boundary Γ, which is subjected to body forces f and to surface tractions g on the free portion Γ_t of Γ. The constrained boundary Γ_u is fixed and $\Gamma_u \cup \Gamma_t = \Gamma$, $\Gamma_u \cap \Gamma_t = \emptyset$. Let $\dot{\mathbf{u}} = [\dot{u} \quad \dot{v}]^T$ be the velocity or flow fields that belong to a space Y of kinematically admissible velocity fields, where $\dot{u}$ and $\dot{v}$ are the velocity components in the x- and y-directions respectively.

The external work rate associated with a virtual plastic flow $\dot{\mathbf{u}}$ is expressed in linear form as

$$F(\dot{\mathbf{u}}) = \int_\Omega \mathbf{f}^T \dot{\mathbf{u}} \, \mathrm{d}\Omega + \int_{\Gamma_t} \mathbf{g}^T \dot{\mathbf{u}} \, \mathrm{d}\Gamma \tag{1}$$

If $C = \{\dot{\mathbf{u}} \in Y | \; F(\dot{\mathbf{u}}) = 1\}$, then the collapse load multiplier λ can be determined by solving the following mathematical programming problem

$$\lambda^+ = \min_{\dot{\mathbf{u}} \in C} \int_\Omega D(\dot{\boldsymbol{\epsilon}}) \, \mathrm{d}\Omega \tag{2}$$

where strain rates $\dot{\boldsymbol{\epsilon}}$ are given by

$$\dot{\boldsymbol{\epsilon}} = \begin{bmatrix} \dot{\epsilon}_{xx} \\ \dot{\epsilon}_{yy} \\ \dot{\gamma}_{xy} \end{bmatrix} = \nabla \dot{\mathbf{u}} \tag{3}$$

and where the differential operator ∇ is given by

$$\nabla = \begin{bmatrix} \frac{\partial}{\partial x} & 0 \\ 0 & \frac{\partial}{\partial y} \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} \end{bmatrix} \tag{4}$$

The plastic dissipation $D(\dot{\boldsymbol{\epsilon}})$ is defined by

$$D(\dot{\boldsymbol{\epsilon}}) = \max_{\psi(\boldsymbol{\sigma}) \leqslant 0} \boldsymbol{\sigma} : \dot{\boldsymbol{\epsilon}} \equiv \boldsymbol{\sigma}_\epsilon : \dot{\boldsymbol{\epsilon}} \tag{5}$$

in which $\boldsymbol{\sigma}$ represents the admissible stresses contained within the convex yield surface and $\boldsymbol{\sigma}_\epsilon$ represents the stresses on the yield surface associated with any strain rates $\dot{\boldsymbol{\epsilon}}$ through the plasticity condition.

In the framework of a limit analysis problem, only plastic strains are considered and are assumed to obey the normality rule

$$\dot{\boldsymbol{\epsilon}} = \dot{\boldsymbol{\mu}} \frac{\partial \psi}{\partial \boldsymbol{\sigma}} \tag{6}$$

where the plastic multiplier $\dot{\boldsymbol{\mu}}$ is non-negative and the yield function $\psi(\boldsymbol{\sigma})$ is convex. In this study the von Mises failure criterion is used (which is equivalent to the Tresca criterion in plane strain [5]). Thus

$$\psi(\boldsymbol{\sigma}) = \sqrt{\frac{1}{4}(\sigma_{xx} - \sigma_{yy})^2 + \sigma_{xy}^2} - \sigma_0 \tag{7}$$

where σ_0 is the yield stress.

Then the power of dissipation can be formulated as a function of strain rates as [5]

$$D(\dot{\boldsymbol{\epsilon}}) = \sigma_0 \sqrt{\dot{\boldsymbol{\epsilon}}^T \boldsymbol{\Theta} \dot{\boldsymbol{\epsilon}}} \tag{8}$$

where

$$\boldsymbol{\Theta} = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{9}$$

Note that condition (6) acts as a kinematic constraint which confines admissible strain rate vectors. Since the yield surface $\psi(\boldsymbol{\sigma})$ is unbounded, the incompressibility condition $\boldsymbol{\chi}^T \dot{\boldsymbol{\epsilon}} = 0$, where $\boldsymbol{\chi} = [1 \quad 1 \quad 0]^T$, must be introduced to ensure that the plastic dissipation $D(\dot{\boldsymbol{\epsilon}})$ is finite [2,6,28,29].

3. EFG discretization of kinematic formulation

3.1. Moving least squares approximation

By using the moving least squares technique [13,30], which is one of the most frequently used approximations in meshless methods, approximations of the displacement (or displacement rate) fields can be expressed as

$$\mathbf{u}^h(\mathbf{x}) = \begin{bmatrix} u^h \\ v^h \end{bmatrix} = \sum_{I=1}^{n} \Phi_I(\mathbf{x}) \begin{bmatrix} u_I \\ v_I \end{bmatrix} \tag{10}$$

in which

$$\Phi_I(\mathbf{x}) = \mathbf{p}^T(\mathbf{x}) \mathbf{A}^{-1}(\mathbf{x}) \mathbf{B}_I(\mathbf{x}) \tag{11}$$

$$\mathbf{A}(\mathbf{x}) = \sum_{I=1}^{n} w_I(\mathbf{x}) \mathbf{p}(\mathbf{x}_I) \mathbf{p}^T(\mathbf{x}_I) \tag{12}$$

$$\mathbf{B}_I(\mathbf{x}) = w_I(\mathbf{x}) \mathbf{p}(\mathbf{x}_I) \tag{13}$$

where n is the number of nodes; $\mathbf{p}(\mathbf{x})$ is a set of basis functions and $w_I(\mathbf{x})$ is a weight function associated with node I. In this work, an isotropic quartic spline function is used, which is given by

$$w_I(\mathbf{x}) = \begin{cases} 1 - 6s_I^2 + 8s_I^3 - 3s_I^4 & \text{if } s_I \leqslant 1 \\ 0 & \text{if } s_I > 1 \end{cases} \tag{14}$$

with $s_I = \frac{\|\mathbf{x} - \mathbf{x}_I\|}{R_I}$, where R_I is the support radius of node I and determined by

$$R_I = \beta \cdot h_I \tag{15}$$

where β is the dimensionless size of influence domain and h_I is the nodal spacing when nodes are distributed regularly, or the maximum distance to neighbouring nodes when nodes are distributed irregularly; further details can be found in [15]. In the next section, a technique will be presented that allows the required order of differentiation to be reduced by one, with the consequence that there is no need to calculate shape function derivatives for the stabilized EFG formulation.

3.2. Strain smoothing stabilization

A strain smoothing method was firstly presented in [31] for regularization of material instabilities. The strain smoothing method was then modified for stabilization of nodal integration by [24]

$$\tilde{\epsilon}_{ij}^h(\mathbf{x}_J) = \int_{\Omega_J} \epsilon_{ij}^h(\mathbf{x}) \varphi(\mathbf{x}, \mathbf{x} - \mathbf{x}_J) \, \mathrm{d}\Omega \tag{16}$$

where $\tilde{\epsilon}_{ij}^h$ is the smoothed value of strains ϵ_{ij}^h at node J, and φ is a distribution (or smoothing) function that has to satisfy the following properties [31,32]