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A topological approach to delay aversion[☆]

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ABSTRACT

A decision maker is to choose between two different amounts of money, with the smaller one available at an earlier period. Then she is *long-term delay averse* if she chooses the smaller and earlier extra amount whenever the bigger one is delivered sufficiently far in the future. In this paper we study new topologies on l^∞ which "discount" the future consistently with the notion of long-term delay aversion. We compare these topologies with other topologies that have the property of representing impatient, or patient, preferences. Our results bear relevance on the theory of infinite-dimensional general equilibrium and with the works that consider bubbles as the pathological (not countably additive) part of a charge. Finally we develop a notion of more long-term delay aversion and we compare it with the concepts studied by Benoît and Ok (2007).

1. Introduction

One of the standard assumptions made in most economic models is that agents have preferences for advancing the time of future satisfaction. This behaviour is commonly known with the term of impatience.

This paper studies the preferences of a Decision Maker (DM) over infinite flows of income. An alternative interpretation is to think not of an agent but of different generations living in different ages. In this case we will talk about a Social Planner who has preferences over infinite streams of wealth (each period represents the wealth of one generation). In both situations, the natural framework to model this infinite horizon problem is the study of preferences over the set l^∞ of bounded, real-valued sequences.

The classical way of describing impatient preferences is to use the discounted sum of utilities. If the DM is facing a monetary flow $(x_0, x_1, \ldots)$ then she evaluates it through the functional:

$$U(x_0, x_1, \ldots) = \sum_{t=0}^{\infty} \delta(t)u(x_t).$$

The function $u : \mathbb{R} \to \mathbb{R}$ is an instantaneous utility function that represents the utility derived by using a certain amount of income. The decreasing function $\delta : \mathbb{N} \to (0, 1]$ is called discount function and it represents the willingness of the DM to anticipate future consumption. Often, the function $\delta(t) = \delta^t$ (where δ is a real number in the interval $(0, 1)$) and the model is called the exponential discounted utility model. The exponential discounted utility model was proposed back in the thirties in a seminal paper of Samuelson (1937). A further boost to its popularity was given by Koopmans (1960) who showed that the model could be derived from a set of plausible axioms. Since then, it has become the standard treatment of impatient behaviours in economics.

Departures from the classical discounted utility model are present in the literature of mathematical economics. These deviations consider functionals different from the discounted sum of utilities presented above. For instance, Chateauneuf and Ventura (2013) use the Choquet integral in order to analyse different kinds of impatient behaviours. Marinacci (1998) characterizes complete patience through the MaxMin model and Rébillé (2007) considers patience in the Choquet model. Finally in Bastianello and Chateauneuf (2016) a concept called long-term delay aversion was introduced and analysed in both the Choquet and MaxMin models.

This paper does not follow any of the approaches aforementioned but instead considers a topological approach, pioneered by Brown and Lewis (1981). We do not specify any utility function for the DM, but we rather focus on the continuity of her preferences with respect to a suitable topology. The topology considered makes the DM "discount" the future in a way consistent with the notion of long-term delay aversion proposed in Bastianello and Chateauneuf (2016). Notice that the choice of the topology over the infinite dimensional space l^∞ is relevant for its behavioural implication. Continuity is not a technical requirement, it is, in fact, a behavioural assumption. Mas-Colell and Zame (1991) put it very sharply:

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"It should be stressed that the choice of the topology can only be dictated by economic, rather than mathematical, considerations".

Instead of assuming discounted utility, we start from the description of long-term delay averse preferences. Long-term delay aversion means the following. Suppose that an agent has to choose between two extra payments of, say, 1000$ and 10 000$. The 1000$ are paid on a fixed date whereas the 10 000$ will be paid later. We believe that, if the second and bigger payment is done sufficiently far in the future, then she will choose the first one. More formally, let us consider a DM who is supposed to receive two additional amounts of income or consumption good, a and b, with $0 < a \leq b$, delivered respectively in periods n_0 and n with $n_0 < n$. Then she is long-term delay averse if she prefers a over b if n is sufficiently big.

After presenting the main definition, we consider two Hausdorff locally convex topologies that represent a future-disliking behaviour consistent with long-term delay aversion. The key idea is that a suitable topology should make a cash flow which pays one unit of income in the nth period very close to the cash flow paying zero at all periods, provided that n is big enough. Such a property could be rephrased as "the far future is negligible".

Endowed with such topologies we proceed comparing them with the strong and weak myopic topologies introduced by Brown and Lewis (1981). These topologies are fundamental in economics and specifically in the theory of general equilibrium in infinite dimension, see Mas-Colell and Zame (1991). Roughly speaking, we find that the long-term delay averse topologies are finer than the myopic topologies. This implies that it is easier to be long-term delay averse rather than myopic and therefore, it is possible to have preference for advancing the time future satisfaction and still an equilibrium may fail to exist. Such a result clarifies a paper of Araujo (1985), where the author shows that impatience is a necessary condition to insure the existence of an equilibrium in an infinite dimensional setting. Our results show that DMs should be *enough* impatient to get an equilibrium.

Next, we study the property of the topological dual of l^∞ when paired with the long-term delay averse topologies. Dual spaces play a major role in general equilibrium since the equilibrium prices are functionals belonging to the dual space. Interestingly, we find that the dual space is bigger than the one obtained with the topologies usually considered. This entails the possibility of having bubbles (in the sense of Gilles and LeRoy, 1992) even when agents show a form of impatience.

As a dividend, we obtain a new characterization of the space *ba* of bounded charges. This space is the dual of l^∞ when paired with a particular long-term delay averse topology.

We conclude the paper with a section devoted to the comparison of two long-term delay averse DMs. We develop a notion of more long-term delay aversion coherent with the one of long-term delay aversion. Finally we discuss the relation of our notion with the work of Benoît and Ok (2007). Loosely speaking, we find that our definition of more long-term delay aversion is weaker than the concept of more delay aversion and stronger than the concept of more impatience studied by Benoît and Ok (2007).

The paper is organized as follows. Section 2 presents some preliminary notions. Sections 3 and 4 present and analyse a long-term delay averse topology and a long-term delay averse topology with a monotone base respectively. Section 5 studies the notion of more long-term delay aversion.

2. Preliminary notions

We study the preferences of a DM over the space l^∞ of real-valued bounded sequences. The generic elements of l^∞ are denoted as $\mathbf{x}, \mathbf{y}$, etc. and are considered as infinite streams of income. The pth element of sequence $\mathbf{x}$ is denoted equivalently x_p or $\mathbf{x}(p)$. Clearly, the set $\mathbb{N}$ of natural numbers represents time.

The sum between two sequences and the multiplication by a scalar correspond to the pointwise sum and multiplication, meaning that if $\mathbf{x}, \mathbf{y} \in l^\infty$ and $\lambda \in \mathbb{R}$ then $\mathbf{x}+\mathbf{y} = (x_0+y_0, x_1+y_1, \ldots)$ and $\lambda\mathbf{x} = (\lambda x_0, \lambda x_1, \ldots)$. The symbol 1_A denotes the indicator function of the set $A \subseteq \mathbb{N}$, i.e. $1_A(n) := \begin{cases} 1 & \text{if } n \in A \\ 0 & \text{if } n \in A^c. \end{cases}$ Therefore 1_A is the sequence with $1_A(p) = 1$ if $p \in A$ and $1_A(p) = 0$ if $p \notin A$ and 1_k the sequence with all the elements equal to 0, but the element k which is equal to 1. Hence given a sequence $\mathbf{x}$, the sequence $\mathbf{x}+a1_k$ denotes the sequence $\mathbf{y}$ such that $y_k = x_k + a$ and $y_n = x_n$ for all $n \neq k$.

A vector space X is an *ordered vector space* with an order $\geq$ if X is partially ordered by $\geq$ and if for every $\mathbf{x}, \mathbf{y}, \mathbf{z} \in X$ and every real number $\lambda \geq 0$, $\mathbf{x} \geq \mathbf{y}$ implies $\mathbf{x}+\mathbf{z} \geq \mathbf{y}+\mathbf{z}$ and $\mathbf{x} \geq 0$ implies $\lambda\mathbf{x} \geq 0$. The space we are considering, l^∞, comes equipped with a natural order. We write $\mathbf{x} \geq \mathbf{y}$ when $x_k \geq y_k \, \forall k$, $\mathbf{x} \gg \mathbf{y}$ when $x_k > y_k \, \forall k$ and $\mathbf{x} > \mathbf{y}$ when $x_k \geq y_k \, \forall k$ with a strict inequality for at least one k. A sequence is non-negative if $\mathbf{x} \geq 0$ and l^∞_+ denotes the positive orthant of l^∞ i.e. $l^\infty_+ := \{\mathbf{x} \in l^\infty : \mathbf{x} \geq 0\}$.

Let X be an ordered vector space. A *seminorm* on X is a function $p : X \to \mathbb{R}$ such that $\forall \mathbf{x}, \mathbf{y} \in X$ and $\forall \alpha \in \mathbb{R}$, (i) $p(\mathbf{x}+\mathbf{y}) \leq p(\mathbf{x})+p(\mathbf{y})$ and (ii) $p(\alpha\mathbf{x}) = |\alpha| p(\mathbf{x})$. If moreover $p(\mathbf{x}) = 0$ if and only if $\mathbf{x} = 0$, then p is called a *norm*. A *locally convex topology* is a topology generated by a family of seminorms. We say that p is a *monotone seminorm* if $0 \leq \mathbf{y} \leq \mathbf{x} \Rightarrow p(\mathbf{y}) \leq p(\mathbf{x})$. A subfamily of seminorms $\mathcal{Q}$ is said to be a *base* for a family of seminorms $\mathcal{P}$ if for every $p \in \mathcal{P}$ there is $q \in \mathcal{Q}$ and $c > 0$ s.t. $p(\mathbf{x}) < cq(\mathbf{x})$ for every $\mathbf{x}$. In this case we say that every seminorm p in $\mathcal{P}$ is dominated by a seminorm in $\mathcal{Q}$. A topology is said to be a *locally convex topology with a monotone base* if the associated family of seminorms has a monotone base.

Regarding convergence of sequences or nets we use the following notation. If $\{a_n\}_{n\in\mathbb{N}}$ is a sequence of real numbers, $a_n \to_n l$ means that the sequence converges to the real number $l \in \mathbb{R}$. If $\{\mathbf{x}_\lambda\}_{\lambda\in\Lambda}$ is a net of elements of a set X endowed with a topology $\mathcal{T}$, then $\mathbf{x}_\lambda \xrightarrow{\mathcal{T}}_\lambda \mathbf{x}$ means that the net converges to the element $\mathbf{x}$ in the topology $\mathcal{T}$. If $\mathcal{T}$ is a locally convex topology generated by a family of seminorms $\{p_\alpha, \alpha \in A\}$ then $\mathbf{x}_\lambda \xrightarrow{\mathcal{T}}_\lambda \mathbf{x}$ if and only if $p_\alpha(\mathbf{x}_\lambda - \mathbf{x}) \to_\lambda 0$ for every $\alpha \in A$ (see Aliprantis and Border, 2006, Lemma 5.76). Sometimes we may write only $\mathbf{x}_n \to \mathbf{x}$ for convergence of sequences (or $\mathbf{x}_\lambda \to \mathbf{x}$ for convergence of nets) when no confusion can arise about the index and the topology that we are considering.

The symbol $\mathcal{T}_\infty$ designates the sup-norm topology on l^∞, that is the topology generated by the supremum norm $\|\mathbf{x}\|_\infty = \sup_k |x_k|$. When l^∞ is endowed with a particular topology $\mathcal{T}$, its *(topological) dual* with respect to $\mathcal{T}$ is the set of $\mathcal{T}$-continuous linear functions on l^∞ and it is denoted $(l^\infty, \mathcal{T})^*$. Let $\mathcal{T}_1$ and $\mathcal{T}_2$ be two topologies on l^∞. If $\mathcal{T}_1 \subseteq \mathcal{T}_2$ we say that $\mathcal{T}_1$ is *weaker (or coarser)* than $\mathcal{T}_2$ or that $\mathcal{T}_2$ is *stronger (or finer)* than $\mathcal{T}_1$. If additionally $\mathcal{T}_1 \neq \mathcal{T}_2$, we write $\mathcal{T}_1 \subsetneq \mathcal{T}_2$ and we say that $\mathcal{T}_1$ is strictly weaker (or strictly coarser) than $\mathcal{T}_2$ or that $\mathcal{T}_2$ is strictly stronger (or strictly finer) than $\mathcal{T}_1$.

A preference relation $\succsim$ over l^∞ is a complete, reflexive and transitive binary relation, i.e. a weak order. Given a preference relation $\succsim$ we denote its symmetric and asymmetric parts by $\sim$ and $\succ$ respectively. We say that a preference relation over l^∞ is *monotone* if $\mathbf{x} \geq \mathbf{y}$ implies $\mathbf{x} \succsim \mathbf{y}$ and *strongly monotone* if $\mathbf{x} > \mathbf{y}$ implies $\mathbf{x} \succ \mathbf{y}$. A preference relation $\succsim$ over l^∞ is *continuous with respect to a topology* $\mathcal{T}$ if the sets of the form $\{\mathbf{x} | \mathbf{x} \succ \mathbf{y}\}$ and $\{\mathbf{x} | \mathbf{y} \succ \mathbf{x}\}$ are $\mathcal{T}$-open for every $\mathbf{y} \in l^\infty$.

Given a set X and a field $\mathcal{F}$ of its subsets, a set function $\mu : \mathcal{F} \to \mathbb{R}$ is called a *charge* if (i) $\mu(\emptyset) = 0$ and (ii) if $A, B \in \mathcal{F}$ and $A \cap B = \emptyset$ then $\mu(A \cup B) = \mu(A) + \mu(B)$. A charge μ is said to be *bounded* if $\sup\{|\mu(F)| : F \in \mathcal{F}\} < +\infty$. If $\mu(F) \geq 0$ for every $F \in \mathcal{F}$ then μ is said to be *positive*. If, given a sequence of sets $\{A_n\}_n$ such that $\cup_n A_n \in \mathcal{F}$ and $A_i \cap A_j = \emptyset$ for $i \neq j$,