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Growth effects of annuities and government transfers in perpetual youth models[☆]

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ABSTRACT

We show that in overlapping generations endogenous growth models with uncertain lifetime, the introduction of government transfers always increases economic growth by crowding out the private annuity market and increasing accidental bequests. In particular, if the government imposes a flat-rate consumption tax (which is neutral to the consumption–saving margin), uses part of the tax revenue for unproductive purposes, and rebates the rest equally across agents as a lump-sum transfer, the economy grows faster and improves the welfare of future generations.

1. Introduction

Suppose that the government imposes some tax, uses part of the tax revenue for unproductive purposes, and rebates the rest to agents. Would this policy increase or decrease economic growth? Intuition tells us that the growth rate will decrease, since resources are wasted after all. In this paper, we show that this intuition is not generally correct: in perpetual youth models, if the tax does not directly affect growth (which is true for flat-rate consumption tax), then this redistribution policy unambiguously *increases* economic growth by crowding out the private annuity market and increasing accidental bequests.

This paper studies the effect of annuities and transfers on economic growth in perpetual youth models (Yaari, 1965; Blanchard, 1985), where agents die at a constant rate and new agents are born at the same rate. We show that perpetual youth models with annuities have three forces that modify economic growth relative to the benchmark economy with infinitely-lived agents: (i) impatience $(-)$, (ii) effective risk-free rate $(+)$, and (iii) accidental bequests $(-)$, where $(\pm)$ denote the positive or negative effect on growth. The first negative effect always dominates the second positive effect, and hence the growth rate unambiguously decreases in perpetual youth models relative to the benchmark case, which is similar to the well-known result that the steady state capital is lower in perpetual youth models with decreasing returns to scale (Blanchard, 1985). However, when agents receive government transfers in this economy, it reduces the third negative effect while leaving the first two effects unchanged. Consequently, the introduction of government transfers in perpetual youth models unambiguously increases economic growth.

The intuition for this somewhat surprising result is as follows. Since agents die at a constant rate $\delta > 0$, in the absence of government transfers, agents pledge their capital (wealth) to insurance companies to obtain annuities at premium δ. In the presence of transfers, part of the agents' wealth is the "government bond" (a claim to future transfers), but because the transfer is given only to agents that are alive, this government bond is not pledgeable to insurance companies. Provided that the tax instruments to finance the transfers do not directly affect growth, the introduction of government transfers crowds out the private annuity market, increases accidental bequests, and leads to higher economic growth. This is the case when we consider a flat-rate consumption tax.

This paper is related to two strands of literature. The first is the large literature on taxation and growth. In this literature, researchers typically consider dynamic models that feature some inefficiencies such as externalities, public goods, or incomplete markets, and study the effect of taxation on growth and welfare. Examples are human capital formation (Lucas, 1988; King and Rebelo, 1993), provision of productive public goods (Barro, 1990; Jones et al., 1993; Hatfield, 2015), saving behavior under uninsured idiosyncratic risk (Aiyagari, 1994; Angeletos, 2007), bequest motive (Ihori, 2001), and political economy (Jaimovich and Rebelo,

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2016), among others.[1] However, sources of inefficiencies are not necessary to make the study of taxation and growth interesting. For example, Jones and Manuelli (1992) show that in an overlapping generations model with finite lives and convex technologies, (i) the consumption path in a laissez-faire economy is always bounded, but (ii) taxing the old and subsidizing the young may sustain growth. The intuition is that taxing the old makes future consumption more expensive and induces the young to save. Compared to this literature, our results are complementary since we only consider flat-rate consumption tax (with a single good), which does not directly affect growth since it is neutral to the consumption–saving margin (Stokey and Rebelo, 1995).[2]

Our paper is also related to the literature that employs perpetual youth models (Yaari, 1965; Blanchard, 1985), which are convenient for studying intergenerational issues in a tractable way and introducing stationarity in heterogeneous-agent models. Recent applications are asset pricing (Gârleanu et al., 2012; Gârleanu and Panageas, 2015), retirement (Prettner and Canning, 2014), and power laws in income and wealth distributions (Toda, 2014; Toda and Walsh, 2015; Benhabib et al., 2016; Gabaix et al., 2016), among others. Although several papers have studied the growth effects of taxation and/or annuities in perpetual youth models,[3] our mechanism that government transfers increase growth by crowding out the annuity market does not seem to be known. The closest result to ours is Petrucci (2002), who shows that consumption tax and rebates increase economic growth in perpetual youth models. However, his model contains special features such as production externality, log utility, and perfect annuity market, so it is not clear whether the results are general. Most importantly, Petrucci (2002) does not identify the key mechanism that government transfers increase growth by crowding out the annuity market.

2. Growth in perpetual youth models

In this section we show how annuities and transfers affect economic growth in perpetual youth models. We first consider the benchmark economy with infinitely-lived agents, and then introduce annuities and transfers when agents enter/exit the economy.

2.1. Benchmark economy

The model is a continuous-time endogenous growth model (*AK* model) with a continuum of agents and a government. At time $t = 0$, there is a continuum of identical, infinitely-lived agents with mass 1, each endowed with one unit of capital.

Agents have identical, additively separable utility function with constant elasticity of substitution

$$U_t = \int_0^\infty e^{-\beta s} \frac{c_{t+s}^{1-1/\varepsilon}}{1-1/\varepsilon} ds, \tag{2.1}$$

where $\beta > 0$ is the time preference rate, $\varepsilon > 0$ is the elasticity of intertemporal substitution, and c_t is consumption at time t. As usual, the case $\varepsilon = 1$ corresponds to the log utility.

Capital can be either consumed or invested in a saving technology that yields an exogenous, risk-free return μ.[4] Alternatively, we can think of a small open economy that has access to a risk-free asset with elastic supply, whose rate is set by international investors. Thus an agent's objective is to maximize the utility (2.1) subject to the budget constraint

$$dw_t = (\mu w_t - c_t)dt, \tag{2.2}$$

where w_t is wealth. This problem is a standard Merton (1971)-type optimal consumption–saving problem. The optimal consumption rule is $c_t = m_0 w_t$,[5] where the marginal propensity to consume (m_0) is given by

$$m_0 = \varepsilon\beta + (1-\varepsilon)\mu. \tag{2.3}$$

The growth rate of the individual wealth α_0 (as well as the growth rate of aggregate wealth g_0 since it is a representative-agent model) is given by

$$\alpha_0 = g_0 = \mu - m_0 = \varepsilon(\mu - \beta). \tag{2.4}$$

As is well known, whether the economy grows or shrinks over time depends on whether or not the interest rate μ exceeds the time preference rate β.

2.2. Overlapping generations economy with annuities

Next, instead of assuming infinitely-lived agents, suppose that agents are born and die at constant Poisson rate $\delta > 0$, as in Yaari (1965) and Blanchard (1985). In addition to the agents, there are perfectly competitive insurance companies that offer annuities. Since agents die at a constant rate δ, the insurance premium is also δ. This means that while agents are alive, for each unit of annuity contract held, the agents receive $\delta\Delta t$ during a small time interval of length Δt. When they die, they pay 1 to the annuity company, which breaks even.

To see how the introduction of annuities affects economic growth, it is convenient to consider an economy with imperfect annuities: following Hansen and İmrohoroğlu (2008), agents can only pledge a fraction $0 \le \lambda \le 1$ of their capital for the annuity contracts. When an agent dies, his heir inherits the remaining fraction $1-\lambda$.

The solution to the individual problem is similar to the benchmark case. Since agents die at rate $\delta > 0$, it increases the effective discount factor from β to $\beta + \delta$. Since agents can receive annuities at rate δ on fraction of wealth λ, the effective risk-free rate faced by individuals becomes $\mu + \delta\lambda$. By (2.3) and (2.4), the propensity to consume out of wealth and the individual growth rate become

$$m_1 = \varepsilon(\beta+\delta) + (1-\varepsilon)(\mu+\delta\lambda) = m_0 + \varepsilon\delta(1-\lambda) + \delta\lambda, \tag{2.5a}$$
$$\alpha_1 = \varepsilon(\mu + \delta\lambda - \beta - \delta) = \alpha_0 - \varepsilon\delta(1-\lambda), \tag{2.5b}$$

respectively.

To derive the growth rate of the aggregate economy, consider what happens to an agent with wealth w_t between a short time period Δt. If the agent survives (which occurs with probability $1 - \delta\Delta t$), then the wealth grows at rate α_1 in (2.5b), so it becomes

[1] Although our paper is purely theoretical, for empirical evidence on the relation between taxation and growth, see for example Engen and Skinner (1996) and Lee and Gordon (2005).

[2] Other forms of taxes may mechanically affect growth by intervening in the intra- and intertemporal choices, such as differential tax rates on production factors (Easterly, 1993) or capital income taxation (Uhlig and Yanagawa, 1996).

[3] See, for example, Hu (1999), Reinhart (1999), Heijdra and Ligthart (2000), Hansen and İmrohoroğlu (2008), Heijdra and Mierau (2010, 2012), and Heijdra et al. (2014).

[4] This assumption is only for simplicity. We obtain the same results with stochastic returns. If capital evolves according to a geometric Brownian motion with volatility σ, all results go through by changing μ to $\mu - \frac{\gamma\sigma^2}{2}$, where $\gamma > 0$ is the relative risk aversion of the agents. In that case one needs to consider the continuous-time analogue of the Epstein–Zin utility (Svensson, 1989; Duffie and Epstein, 1992).

[5] In order for this to be the solution, we need the parameter restriction $\varepsilon\beta + (1-\varepsilon)\mu > 0$. Otherwise, the transversality condition $\lim_{t\to\infty} e^{-\beta t} U_t = 0$ is violated, and there is no solution. The transversality condition is sufficient for optimality. See, for example, the discussion of the verification theorem in Chang (2004, pp. 122–125).