Computers and Structures 106–107 (2012) 68–80

Contents lists available at SciVerse ScienceDirect

Computers and Structures

journal homepage: www.elsevier.com/locate/compstruc

An optimized BEM–FEM iterative coupling algorithm for acoustic–elastodynamic interaction analyses in the frequency domain

D. Soares Jr. [a,*], L. Godinho [b]

[a] Structural Engineering Department, Federal University of Juiz de Fora, CEP 36036-330 Juiz de Fora, MG, Brazil
[b] CICC, Department of Civil Engineering, University of Coimbra, 3030-788 Coimbra, Portugal

ARTICLE INFO

Article history:
Received 9 January 2012
Accepted 23 April 2012
Available online 17 May 2012

Keywords:
Acoustics
Elastodynamics
Boundary Element Method
Finite Element Method
Frequency domain analysis
Optimal relaxation parameters

ABSTRACT

In this work, a coupled BEM–FEM strategy for the analysis of fluid–solid interaction problems in the frequency domain is presented. Here, acoustic fluids are modelled by the BEM, whereas elastodynamic solids are discretized by the FEM. The fluid–solid coupling is carried out by an optimized iterative procedure. This coupling technique allows independent discretizations to be efficiently employed for both Boundary and Finite Element Methods, without any requirement of matching nodes at the fluid/solid common interfaces. Optimal relaxation parameters are computed, in order to ensure the convergence of the iterative procedure, properly dealing with the frequency domain wave propagation ill-posed problem.

1. Introduction

Numerical analysis of fluid–solid coupled systems is a complex task, requiring proper treatment of sub-domains in which different physical phenomena are involved, as well as suitable numerical modelling of wave propagation across arbitrary shaped interfaces.

In the present work, two distinct methods are considered in order to numerically discretize the different sub-domains of the fluid–solid coupled model, namely: the Boundary Element Method (BEM) and the Finite Element Method (FEM). As it is well known, the FEM is well suited for modelling inhomogeneous and anisotropic solids, as well as for dealing with non-linear behaviour. The BEM, on the other hand, is an appropriate numerical tool to discretize acoustic fluids with infinite extension and/or high gradient concentrations. Thus, coupling boundary and finite element procedures allows the combination of several advantages, which is beneficial for fluid–solid interaction analysis.

Considering fluid–solid interaction modelling, most of the BEM–FEM coupling algorithms [1–9] are formulated in a way that a coupled system of equations is established, which afterwards has to be solved using a standard direct solution scheme. Such a procedure leads to several problems with respect to efficiency, accuracy and flexibility. First, the coupled system of equations has a banded symmetric structure only in the FEM part, while in the BEM part it is non-symmetric and fully populated. Consequently, for its solution the optimized solvers usually used by the FEM cannot be employed anymore, which leads to rather expensive calculations with respect to computer time. Second, fluid and solid media usually have quite different physical properties, resulting in bad-conditioned coupled matrices when standard coupling procedures are considered. This may affect the accuracy of the methodology, providing misleading results. Third, the standard coupling methodology does not allow independent discretization for each sub-domain of the model, requiring matching nodes at common interfaces, which drastically affects the flexibility and versatility of the technique.

In order to evade these drawbacks, iterative coupling procedures have been recently presented, taking into account time domain fluid–solid interacting analyses, considering boundary/finite element formulations [10–12]. As it has been reported, iterative coupling approaches allow BEM and FEM sub-domains to be analysed separately, leading to smaller and better-conditioned systems of equations (different solvers, suitable for each sub-domain, may be employed). Moreover, a small number of iterations is required for the algorithm to converge and the matrices related to the smaller governing systems of equations do not need to be treated (inverted, triangularized etc.) at each iterative step, providing an efficient methodology. As a matter of fact, in time domain analyses, iterative coupling procedures have been reported as effective techniques taking into account several wave propagation problems, being not restricted to fluid–solid applications [13–15]. In non-transient problems, iterative coupling methodologies have

* Corresponding author.
E-mail addresses: delfim.soares@ufjf.edu.br (D. Soares Jr.), lgodinho@dec.uc.pt (L. Godinho).

0045-7949/$ - see front matter © 2012 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.compstruc.2012.04.010

also been reported as appropriate numerical tools, being several works presented on the topic, mostly considering potential and elastostatic problems [16–18].

In frequency domain analyses, although rarely, iterative coupling procedures have been reported in the literature, mostly considering acoustic–acoustic coupling [19,20]. As it has been reported, frequency domain wave propagation analyses usually give rise to ill-posed problems and, in these cases, the convergence of the iterative coupling algorithm can be either too slow or unachievable. This is the case in fluid–solid interacting models and, as discussed in this work, convergence can be hardly achieved if no special procedure is considered, especially if higher frequencies are focused. In order to deal with this ill-posed problem and ensure convergence of the iterative coupling algorithm, an optimal iterative procedure is adopted here, with optimal relaxation parameters being computed at each iterative step. As it is described along the paper, the introduction of these optimal relaxation parameters allows the iterative coupling technique to be very effective in the frequency domain, ensuring convergence at a low number of iterative steps.

The paper is organized as follows: first, the governing equations of the physical problem are presented; then, the Boundary and Finite Element Methods are briefly discussed. In the sequence, the iterative coupling technique is described, including the mathematical derivation of the optimization methodology. At the end of the paper, numerical applications are presented, illustrating the accuracy, performance and potentialities of the proposed procedures.

2. Governing equations

In this section, acoustic and elastic wave equations are briefly presented. Each one of these wave propagation models is used to mathematically describe different sub-domains of the global problem. At the end of the section, basic equations concerning the coupling of acoustic and dynamic sub-domains are described.

2.1. Acoustic sub-domains

The acoustic scalar wave equation is given by:

$$(\kappa(X)p(X,t)_{,i})_{,i} - \rho(X)\ddot{p}(X,t) - \nu(X)\dot{p}(X,t) + S(X,t) = 0 \tag{1}$$

where $p(X,t)$ stands for hydrodynamic pressure distribution and $S(X,t)$ for body source terms. Inferior commas (indicial notation is adopted) and over dots indicate partial space ($p_{,i} = \partial p/\partial x_i$) and time ($\dot{p} = \partial p/\partial t$) derivatives, respectively. $\nu(X)$ stands for the viscous damping coefficient, $\rho(X)$ is the mass density and $\kappa(X)$ is the bulk modulus of the medium. In homogeneous media, ν, ρ and κ are constant and the classical Helmholtz wave equation (frequency domain analysis) can be written as:

$$p(X,\omega)_{,ii} + \gamma^2 p(X,\omega) + s(X,\omega) = 0 \tag{2}$$

where $\gamma = \sqrt{\omega^2/c^2 - \mathrm{i}\omega\nu/\kappa}$ stands for the complex wave number, $c = \sqrt{\kappa/\rho}$ is the wave propagation velocity and ω is the frequency. The boundary conditions of the problem are given by:

$$p(X,\omega) = \bar{p}(X,\omega) \quad \text{for} \quad X \in \Gamma_1 \tag{3a}$$

$$q(X,\omega) = p_{,j}(X,\omega)n_j(X) = \bar{q}(X,\omega) \quad \text{for} \quad X \in \Gamma_2 \tag{3b}$$

where the prescribed values are indicated by over bars and $q(X,\omega)$ represents the flux along the boundary whose unit outward normal vector components are represented by $n_j(X)$. The boundary of the model is denoted by $\Gamma(\Gamma_1 \cup \Gamma_2 = \Gamma$ and $\Gamma_1 \cap \Gamma_2 = 0$, where Γ_1 stands for the essential or Dirichlet boundary and Γ_2 stands for the natural or Neumann boundary) and the domain by Ω.

2.2. Elastodynamic sub-domains

The frequency domain elastic wave equation for homogenous media is given by:

$$(c_d^2 - c_s^2)u_j(X,\omega)_{,ji} + c_s^2 u_i(X,\omega)_{,jj} + (\omega^2 - \mathrm{i}\omega\nu)u_i(X,\omega) + b_i(X,\omega) = 0 \tag{4}$$

where $u_i(X,\omega)$ and $b_i(X,\omega)$ stand for the displacement and the body force distribution components, respectively. The notation for space derivatives employed in Eq. (1) is once again adopted. In Eq. (4), c_d is the dilatational wave velocity and c_s is the shear wave velocity, they are given by: $c_d^2 = (\lambda + 2\mu)/\rho$ and $c_s^2 = \mu/\rho$, where ρ is the mass density and λ and μ are the Lamé's constants. ν stands for viscous damping related parameters. Eq. (4) can be obtained from the combination of the following basic mechanical equations (proper to model heterogeneous media):

$$\sigma_{ij}(X,\omega)_{,j} + (\rho(X)\omega^2 - i\omega\rho(X)\nu(X))u_i(X,\omega) + \rho(X)b_i(X,\omega) = 0 \tag{5a}$$

$$\sigma_{ij}(X,\omega) = \lambda(X)\delta_{ij}\varepsilon_{kk}(X,\omega) + 2\mu(X)\varepsilon_{ij}(X,\omega) \tag{5b}$$

$$\varepsilon_{ij}(X,\omega) = \frac{1}{2}(u_i(X,\omega)_{,j} + u_j(X,\omega)_{,i}) \tag{5c}$$

where $\sigma_{ij}(X,\omega)$ and $\varepsilon_{ij}(X,\omega)$ are, respectively, stress and strain tensor components and δ_{ij} is the Kronecker delta ($\delta_{ij} = 1$, for $i = j$ and $\delta_{ij} = 0$, for $i \neq j$). Eq. (5a) is the momentum equilibrium equation; Eq. (5b) represents the constitutive law of the linear elastic model and Eq. (5c) stands for kinematical relations. The boundary conditions of the elastodynamic problem are given by:

$$u_i(X,\omega) = \bar{u}_i(X,\omega) \quad \text{for} \quad X \in \Gamma_1 \tag{6a}$$

$$\tau_i(X,\omega) = \sigma_{ij}(X,\omega)n_j(X) = \bar{\tau}_i(X,\omega) \quad \text{for} \quad X \in \Gamma_2 \tag{6b}$$

where the prescribed values are indicated by over bars and $\tau_i(X,\omega)$ denotes the traction vector along the boundary ($n_j(X)$, as indicated previously, stands for the components of the unit outward normal vector).

2.3. Acoustic–elastodynamic interacting interfaces

On the acoustic–elastodynamic interface boundaries, the dynamic sub-domain normal (normal to the interface) displacements ($u_n(X,\omega)$) are related to the acoustic sub-domain fluxes ($q(X,\omega)$), and the acoustic sub-domain hydrodynamic pressures ($p(X,\omega)$) are related to the dynamic sub-domain normal tractions ($\tau_n(X,\omega)$). These relations are expressed by the following equations:

$$u_n(X,\omega) + 1/(\rho(X)\omega^2)q(X,\omega) = 0 \tag{7a}$$

$$\tau_n(X,\omega) + p(X,\omega) = 0 \tag{7b}$$

where in Eqs. (7a) and (7b) the sign of the different sub-domain outward normal directions is taken into account (outward normal vectors on the same interface point are opposite for each sub-domain). In Eq. (7a), $\rho(X)$ is the mass density of the interacting acoustic sub-domain medium.

3. Numerical modelling

Here, the acoustic fluid sub-domains are analyzed by the Boundary Element Method, whereas the elastodynamic solid sub-domains are discretized by the Finite Element Method. The employed boundary and finite element formulations are briefly described in the subsections that follow.