Journal of Mathematical Economics 70 (2017) 36–44

Contents lists available at ScienceDirect

Journal of Mathematical Economics

journal homepage: www.elsevier.com/locate/jmateco

# On stable rules for selecting committees

Eric Kamwa *

*Université des Antilles, Faculté de Droit et d'Economie de la Martinique, Campus de Schoelcher, F-97275 Schoelcher, France*
*Laboratoire Caraïbéen de Sciences Sociales, LC2S UMR CNRS 8053, France*

ARTICLE INFO

*Article history:*
Received 11 March 2016
Received in revised form
17 January 2017
Accepted 23 January 2017
Available online 22 February 2017

*Keywords:*
Committee
Condorcet
Stable rule
Reinforcement axiom
Pareto criterion

ABSTRACT

A voting rule is said to be *stable* if it always elects a fixed-size subset of candidates such that there is no outside candidate who is majority preferred to any candidate in this set whenever such a set exists. Such a set is called a *Weak Condorcet Committee* (WCC). Four stable rules have been proposed in the literature. In this paper, we propose two new stable rules. Since nothing is known about the properties of the stable rules, we evaluate all the identified stable rules on the basis of some appealing properties of voting rules. We show that they all satisfy the Pareto criterion and they are not monotonic. More, we show that every stable rule fails the reinforcement requirement.



## 1. Introduction

Modern democracies use different voting rules (systems) for electing parliaments or groups (committees) of representatives. The most popular voting rules are, among others, the Plurality rule (used in India, in Great Britain, etc.), the Proportional system (used in Germany, Lebanon, etc.). A committee or a group of representatives is a fixed-size set of alternatives (candidates) chosen from a larger set of contenders. Committees are chosen to fulfill a given purpose and their composition can be subject to some constraints or prerequisites: gender equity, minority representation, quotas and so on. Though a committee meets the prerequisites, it may happen that one wonders about the real legitimacy of this committee since different voting rules may lead to different outcomes. As the legitimacy of a committee does not depend only on its composition but also on the voting rule used, this gives rise to a couple of questions. What is a good committee? Does such a committee exist? What should be a good voting rule for selecting committees? Since the seminal works of Sterne (1871) and Dodgson (1876, 1884, 1885b,a), many political scientists and social choice theorists have tried to suggest how voting rules can be designed or used for selecting committees. See among others the works of Tullock (1967), Hill (1988), Good and Tideman (1976), Chamberlin and Courant (1983), Dummett (1984), Monroe (1995), Brams et al. (2005), Kilgour et al. (2006) and Brams (2008). A recent paper of Elkind et al. (2014) examined the properties of some voting rules in multi-winners context.

According to Gerhlein (1985), one of the prerequisites that can be imposed for committee selection is the fulfillment of the *Condorcet criterion*[1] (Condorcet, 1785). For committee elections to be in line with the Condorcet criterion, Gerhlein (1985) suggested the selection, when it exists, of the fixed-size subset of candidates such that every member majority dominates every non-member: the *Condorcet committee* (à la Gerhlein). Such a set does not always exist in general (Gerhlein, 1985); when it exists it is unique (Good, 1971; Miller, 1980). Gerhlein (1985) computed the likelihood of such a set to exist using Monte Carlo simulations. He found that with four contenders there is a 73.6% chance to end with a Condorcet committee of two members and 82.4% for a three-member committee; with seven contenders, there is a 35% chance to get a Condorcet committee of three members and 31.2% for a four-member committee. The likelihood of a Condorcet committee tends to decrease with the number of contenders and the size of the committee to be elected. Gerhlein (1985) concluded that almost all the well-known voting rules do not always select the Condorcet committee à la Gerhlein when it exists. Up to our knowledge, there are only two voting rules that have been suggested as always selecting the *Condorcet committee* (à la

* Correspondence to: Université des Antilles, Faculté de Droit et d'Economie de la Martinique, Campus de Schoelcher, F-97275 Schoelcher, France.
*E-mail address:* eric.kamwa@univ-antilles.fr.

[1] In one winner-election, this criterion requires that a candidate should be declared as the winner if he defeats each of the other candidates in pairwise comparisons; such a candidate is called the *Condorcet winner*.

http://dx.doi.org/10.1016/j.jmateco.2017.01.008
0304-4068/

Gerhlein) when it exists. These rules suggested by Ratliff (2003) are:

- The *Kemeny–Ratliff rule* (KR), which is an adaptation of the *Kemeny rule*,[2] selects the subset of $g$ candidates with the smallest total margin of loss in pairwise comparisons versus the $m - g$ remaining candidates.
- The *Dodgson–Ratliff rule* (DR) is an adaptation of the Dodgson rule.[3] It selects the subset of $g$ candidates that requires the fewest number of adjacency switches to become a Condorcet committee à la Gerhlein.

Many recent papers have focused on the conditions that guarantee the existence of the *Condorcet committee*. See among others the works of Darmann (2013), Kaymak and Sanver (2003), Kamwa and Merlin (2013) and Elkind et al. (2011, 2015a). What comes out from the results of these authors is that the Condorcet committee seems to be more restrictive as it is hard to get or to find.[4] There is a version of the Condorcet committee that is less demanding and more likely to exist: the *weak Condorcet Committee* (WCC). A WCC is a fixed-size subset of candidates such that none of its members is defeated in pairwise comparisons by an outside candidate. It is obvious that a *Condorcet committee* (à la Gerhlein) is also a WCC; but a WCC is not necessarily a *Condorcet committee* (à la Gerhlein).[5] Given that $g$ is the size of the committee to be elected, the WCC does not exist for some voting profiles while there may exist more than one WCC for some voting profiles.

According to Coelho (2004)[6] a voting rule is said to be *stable* if it always selects a WCC when it exists. Coelho (2004) showed that almost all the well-known voting rules in the social choice literature and even those in use in the real life (such as the Plurality rule and the Borda rule) are not stable. Coelho (2004) concluded that the *Kemeny–Ratliff rule* and the *Dodgson–Ratliff rule* are also stable if they are used for the selection of WCC.[7] Coelho (2004) also suggested two other stable rules:

- The *Minimal Number of External Defeats rule* (NED) which selects the committee(s) of size $g$ for which the number of pairwise comparisons lost by its members is minimal.
- The *Minimal Size of External Opposition rule* (SEO) which is clearly an adaptation of the Maximin rule[8] to committee elections. Given a committee of size $g$, its margin of loss is the highest margin of loss of a candidate in this committee against an outside candidate. The SEO rule elects the committee(s) with the smallest margin of loss.

Coelho (2004) argued that the Kemeny rule, the Dodgson rule and the Maximin rule are not stable when selecting committees by just appointing the best $g$ candidates of these rules. Kamwa (2014) came to a similar conclusion about the *Young rule*.[9] In this paper, we suggest two new stable rules:

- The *Young–Condorcet rule* (YC) which is adapted from the Young rule for committee elections. Given $g$ as the size of the committee to be elected, the YC rule will select the set of $g$ candidates that need the fewest number of deletions of voters to become a WCC.
- The *Minimal Deletion of Candidates rule* (MDC) which selects the set of $g$ candidates that need the fewest number of deletions of candidates to become a WCC.

We have to point out that if we want to select one-member committees with an odd number of voters, the KR rule is equivalent to the Kemeny rule, the DR rule to the Dodgson rule, the SEO rule to the Maximin rule and the YC rule to the Young rule. So, in this paper, our concern will be on committees of at least two members.

Even though four of the stable rules we focus on are adapted from well-known voting rules, nothing is known about their normative properties. Barberà and Coelho (2008) have shown that stability is incompatible with the property of *enlargement consistency*. *Enlargement consistency* requires that whenever a candidate is included in the chosen committee of size $g$, he must also be in the chosen committee of size $g + 1$. As a first step toward a characterization of the whole family of stable rules, we evaluate our stable rules on the basis of some appealing properties of voting rules: the *Condorcet winner* criterion, the *Condorcet loser* criterion, the *Pareto* criterion, the *monotonicity* criterion, the *homogeneity* criterion and the *reinforcement* criterion. All these criteria are defined later.

The rest of the paper is structured as follows: Section 2 sets the framework with basic definitions. The formal definitions of the stable rules are provided in Section 3. In Section 4, we proceed to the evaluation of our stable rules. Section 5 concludes.

## 2. Binary relations and preferences

Let $N$ be the set of $n$ voters ($n \geq 2$) and $A$ the set of $m$ candidates ($m \geq 3$). A binary relation $R$ over $A$ is a subset of the Cartesian product $A \times A$. For $a, b \in A$, if $\{a, b\} \in R$, we note $aRb$ to say "$a$ is at least as good as $b$". $\neg aRb$ is the negation of $aRb$. If we have $aRb$ and $\neg bRa$, we will say "$a$ is better or strictly preferred to $b$". In this case, we write $aPb$ with $P$ denoting the asymmetric component of $R$. The symmetric component of $R$, $I$, is defined by $aIb$ denoting an indifference between $a$ and $b$, *i.e.*, $aRb$ and $bRa$. The preference profile $\pi = (P_1, P_2, \ldots, P_i, \ldots, P_n)$ gives all the linear orders[10] of all the $n$ voters on $A$, where $P_i$ is the strict ranking of a given voter $i$. The set of all the preference profiles of size $n$ on $A$ is denoted by $P(A)^n$. In the sequel, we will simply write, $abc$ to denote that candidate $a$ is ranked before candidate $b$ who is ranked before $c$. A *voting situation* $\tilde{n} = (n_1, n_2, \ldots, n_t, \ldots, n_{m!})$ indicates the number of voters for each linear order such that $\sum_{t=1}^{m!} n_t = n$.

2 Given a preference profile with at least three candidates, the Kemeny rule (Kemeny, 1959; Kemeny and Snell, 1960) operates by computing distances from a given linear order to all the linear orders of the preference profile. The Kemeny ranking is the linear order that minimizes the total distance to the whole profile; the *Kemeny winner* is the candidate at the top of this ranking.

3 The Dodgson rule (Dodgson, 1876) elects the candidate who requires the fewest number of switches (or adjacent switches of candidates) in voters' preferences in order to become the Condorcet winner. By an adjacent switch of $x$ and $y$ we mean swapping them in the given linear order. As shown by Bartholdi et al. (1989) and Hemaspaandra et al. (1997), computing the Dodgson-scores is computationally intractable.

4 Please refer to the work of Darmann (2013).

5 There is also another definition of the *Condorcet committee* suggested by Fishburn (1981); please see the works of Kaymak and Sanver (2003) and Kamwa and Merlin (2013) for the connections between the Fishburn's and the Gerhlein's definitions.

6 See also the paper of Barberà and Coelho (2008).

7 The original KR rule is roughly applied; the DR rule will select the subset of $g$ candidates that requires the fewest number of adjacency switches to make this subset become a WCC. See also the papers of Kamwa (2013a, 2016).

8 Given a voting situation, the *Maximin rule* (also called the *Simpson–Kramer rule* Simpson, 1969; Kramer, 1977 or the *Minimax rule* Young, 1977), first determines the support received by each candidate in every pairwise comparison; the candidate with the greatest minimum support received is the winner. This rule can be traced back to Condorcet (1785) (see also the book of Black, 1958).

9 Suggested by Young (1977), this rule proceeds by deletions of voters. The Young rule elects the candidate(s) who needs the fewest number of deletions of voters to become the Condorcet winner. Computing the Young-scores is computationally intractable: see among others the works of Rothe et al. (2003), Betzler et al. (2010) and Caragiannis et al. (2012).

10 A linear order is a binary relation that is transitive, reflexive, complete and antisymmetric. A binary relation $R$ on $A$ is *transitive* if for $a, b, c \in A$, if $aRb$ and $bRc$ then $aRc$. $R$ is reflexive if for all $a \in A$, one can write $aRa$. $R$ is *complete* if and only if for all $a, b \in A$, we have $aRb$ or $bRa$. $R$ is antisymmetric if for all $a, b \in A$, $aRb \Rightarrow \neg bRa$.