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A dual approach to ambiguity aversion☆

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ABSTRACT

In the present paper, the assumption of monotonicity of Anscombe and Aumann (1963) is replaced by an assumption of monotonicity with respect to first-order stochastic dominance. This yields a representation result where ambiguous distributions of objective beliefs are first aggregated into "equivalent unambiguous beliefs" and then risk preferences are used to compute the utility of these equivalent unambiguous beliefs. Such an approach makes it possible to disentangle uncertainty aversion, related to the processing of information, from risk aversion, related to the evaluation of the equivalent unambiguous beliefs. An application to portfolio choice shows the tractability of the framework and its intuitive appeal.

1. Introduction

The notion of ambiguity aversion is nowadays a central one in economics, almost on a par with risk aversion. There are many available ambiguity models, such as those of Schmeidler (1989); Gilboa and Schmeidler (1989); Klibanoff et al. (2005); Maccheroni et al. (2006). All these ambiguity models, have however in common that they were derived in the horse-roulette setting of Anscombe and Aumann (1963) (hereafter "AA"), and maintain Monotonicity, in the sense introduced by AA, as one of the basic assumptions. This adherence to AA's monotonicity assumption left interesting possibilities unexplored.

The current paper proposes to replace AA's monotonicity assumption by an assumption of monotonicity with respect to first order stochastic dominance. Moreover, a property of tail separability that characterizes AA's subjective expected utility model, but which is relaxed in all the ambiguity models mentioned above, will be maintained. So, as with the previous ambiguity models, we suggest taking a step back from AA's approach. This is indeed required to model ambiguity aversion. The direction of exploration is however different. The result is a decision model where ambiguity aversion resembles a pessimistic form of beliefs aggregation. This model has a number of interesting features. In particular, it accommodates both Ellsberg and Allais paradoxes and affords simple and intuitive results as to the impact of ambiguity aversion.

The contrast between the approach developed in the current paper and the ones that fulfill AA's monotonicity assumption can be explained is simple terms. In AA-monotone models (i.e., models that are monotone in the sense of AA), risk preferences are used to relate subjective distributions of roulette lotteries to subjective distributions of utility levels. Thus, in a first step, Anscombe and Aumann acts are mapped into real-valued acts. Then, in a second step, some specific aggregation procedure is used to evaluate this subjective distribution of utility levels. The diversity of AA-monotone ambiguity models results from the way this second stage aggregation is achieved, but the two-stage modus operandi is maintained as a direct consequence of the monotonicity assumption. The current paper proposes to follow a symmetric path, where a subjective distribution of roulette lotteries is first aggregated to provide a single roulette lottery and then risk preferences are used to estimate the utility of that roulette lottery. Fig. 1 illustrates the difference between the two ways of modeling uncertainty aversion. Further discussion on AA's monotonicity assumption is provided in Section 5.

A noteworthy feature of the dual approach is that it is indistinguishable from the traditional approach when the set of consequences contains only two elements. Indeed, when the set of consequences has only two elements, first-order stochastic dominance generates a complete weak order over roulette lotteries and the AA-monotone and dual routes are completely identical. The dual approach therefore provides models that are just as effective as the AA-monotone models in explaining the results of standard two-outcome Ellsberg's urn experiments.

The dual approach leads, however, to conclusions that typically differ from those of AA-monotone models when there are more than two possible outcomes. With the dual approach, uncertainty

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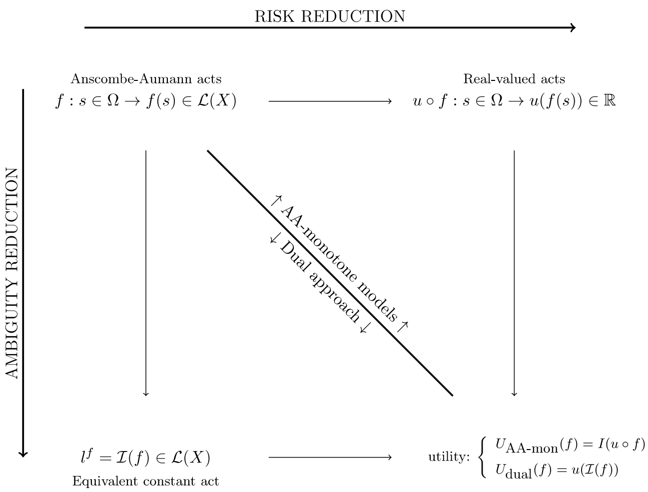

Fig. 1. Two approaches to ambiguity aversion.

aversion only plays a role in the first stage, in which ambiguous beliefs are transformed into equivalent unambiguous beliefs in a way that is independent of risk preferences. Stronger uncertainty aversion leads to using lower unambiguous equivalent beliefs (lower in the sense of first-order stochastic dominance), so that the impact of an increase in uncertainty aversion is identical to that of a change in risk, where one goes from a given distribution to another dominated one. The literature on behavior under risk that has explored the impact of such changes in risk can then be directly used to conclude as to the impact of uncertainty aversion. In consequence, the dual approach affords more straightforward and intuitive comparative statics, with clear-cut predictions that can then be obtained in many cases where AA-monotone models yield ambiguous conclusions. As an example, we will study the impact of uncertainty aversion on portfolio choice.

A couple of other aspects of the dual approach are worth being mentioned. In contrast with all the papers previously cited, the analysis is not restricted to the case where preferences over roulette lotteries are of the expected utility kind. This explains why the model can accommodate both Allais and Ellsberg paradoxes. Moreover, the dual representation is derived without assuming uncertainty aversion. In fact, the mathematical proof does not use the hyperplane separation theorem or the dual representation of convex sets. Instead it relates to the literature on separability, without imposing the assumption of convex preferences. The model can therefore allow for uncertainty loving, or ambiguous uncertainty attitudes. While this paper is not the only one to have these features (see e.g., Segal, 1987), it affords a new way to conceptualize the phenomenon of ambiguity aversion which is viewed as a pessimistic form of beliefs aggregation.

The remainder of the paper is organized as follows. Section 2 introduces the domain of choice and the notation. Section 3 provides the assumptions and the associated representation results. In Section 4, we show how ambiguity attitudes are directly reflected in the averaging of beliefs procedure that characterizes our novel approach. Sections 5 and 6, respectively, discuss AA's monotonicity assumption and the related literature. The application to portfolio choice, which illustrates the interest of the novel approach from an applied point of view, is provided in Section 7.

2. Setting

We consider a connected compact metric set of consequences X. We denote by $\mathcal{L}(X)$ the set of simple lotteries on X. For any simple lottery $l \in \mathcal{L}(X)$, we will denote by $supp(l)$ its support, that is the finite set of elements of X to which l assigns a positive probability. For any $x \in X$, we denote by $\delta_x \in \mathcal{L}(X)$ the lottery that assigns probability 1 to the consequence x.

There is a finite set $S = \{s_1, \ldots, s_N\}$ of states of the world. An act is a function from S into $\mathcal{L}(X)$. We thus denote $\mathcal{F} = \mathcal{L}(X)^N$ the set of acts. This set is endowed with the weak topology. For any act $L = (l_1, \ldots, l_N) \in \mathcal{F}$, we denote by $supp(L)$ the support of L, formally defined by $supp(L) = \cup_{1 \le j \le N} supp(l_j)$. An act is said to be constant if and only if it is an element of the form $(l, \ldots, l)$ for some $l \in \mathcal{L}(X)$. We denote by $\mathcal{F}^c$ the set of constant acts. An act is said to be deterministic if it is of the form $(\delta_x, \ldots, \delta_x)$ for some $x \in X$. Deterministic acts are therefore degenerate constant acts. We use the notation $\Delta_x = (\delta_x, \ldots, \delta_x)$.

For any subset $A \subset X$, and any act $L = (l_1, \ldots, l_N) \in \mathcal{F}$, we denote by $\overrightarrow{Prob}(L \in A) \in [0, 1]^N$ the vector $(Prob(l_1 \in A), \ldots, Prob(l_N \in A))$. A similar notation will be used when "$\in A$" is replaced by some other logical operation. For example, for $x \in X$, we denote by $\overrightarrow{Prob}(L = x)$ the vector $(Prob(l_1 = x), \ldots, Prob(l_N = x)) \in [0, 1]^N$.

The set of acts $\mathcal{F}$ is endowed with a natural mixture operation:

$$\alpha(l_1, \ldots, l_N) + (1 - \alpha)(m_1, \ldots, m_N) = (\alpha l_1 + (1 - \alpha)m_1, \ldots, \alpha l_N + (1 - \alpha)m_N).$$

For any scalar $q \in [0, 1]$, we will denote by $\vec{q} \in [0, 1]^N$ the constant vector $(q, \ldots, q)$. In particular, $\vec{0}$ and $\vec{1}$ will denote the constant vectors $(0, \ldots, 0)$ and $(1, \ldots, 1)$. As is standard, for any $p = (p_1, \ldots, p_N)$ and $p' = (p'_1, \ldots, p'_N)$ in $[0, 1]^N$, the statement $p \ge p'$ is to be understood as $p_j \ge p'_j$ for all $j \in \{1, \ldots, N\}$. The strict inequality $p > p'$ is used to mean that $p \ge p'$ and $p \ne p'$.

The first representation result provided (Theorem 1) does not require preferences over constant acts to fit into the expected utility framework. This gain in generality requires to consider utility representations that take values in $\bar{\mathbb{R}} = \mathbb{R} \cup \{-\infty, +\infty\}$, as in the rank-dependent models of Wakker (1993) and Chateauneuf (1999) for preferences over lotteries. To state our result rigorously, we need to mention some specific conditions of continuity and monotonicity, that we group under the appellation "N-admissibility". For a first reading of the paper, there is no need to bear in mind the details of the definition of N-admissibility below. One may simply think of $N-$admissibility as meaning "continuous and strictly increasing", in a sense that has been fine-tuned to deal with the possibility of infinite valuations, and completed with normalization conditions.

Definition 1 (*N-admissibility*). Assume that X is provided with a weak order $\ge$. The function $v : X \times [0, 1]^N \to \bar{\mathbb{R}}$ is said to be N-admissible if it fulfills the following properties:

1. $v(x, \vec{0}) = 0$ for all $x \in X$.
2. $v(x, p) < +\infty$ except possibly when $x \in max(X)$ and $p = \vec{1}$.[1]
3. $v(x, p) > -\infty$ except possibly when $x \in min(X)$ and $p \ne \vec{0}$.
4. There exists $v_0 \in \mathbb{R} \cup \{-\infty\}$ such that $v(x, p) = v_0$ for all $x \in min(X)$ and all $p \ne \vec{0}$.
5. For any $p \ne \vec{0}$, the function $x \to v(x, p)$ is continuous and strictly increasing.
6. For any p different from $\vec{0}$ and $\vec{1}$, the function $x \to v(x, \vec{1}) - v(x, p)$ is continuous and strictly increasing.
7. For any $x, y \in X$, with $y > x$, the function $p \to v(y, p) + (v(x, \vec{1}) - v(x, p))$ is continuous and strictly increasing.[2]

[1] We use the notation $min(X) = \{x | y \ge x \text{ for all } y \in X\}$ and $max(X) = \{x | x \ge y \text{ for all } y \in X\}$.

[2] If x is a minimal element, the term $(v(x, \vec{1}) - v(x, p))$ may be of the form $-\infty - (-\infty)$. In that case the convention $-\infty - (-\infty) = 0$ should be used. Remark