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Uncertain information structures and backward induction

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ABSTRACT

In everyday economic interactions, it is not clear whether each agent's sequential choices are visible to other participants or not: agents might be deluded about others' ability to acquire, interpret or keep track of data. Following this idea, this paper introduces uncertainty about players' ability to observe each others' past choices in extensive-form games. In this context, we show that monitoring opponents' choices does not affect the outcome of the interaction when every player expects their opponents indeed to be monitoring. Specifically, we prove that if players are rational and there is common strong belief in opponents being rational, having perfect information and believing in their own perfect information, then, the backward induction outcome is obtained regardless of which of her opponents' choices each player observes. The paper examines the constraints on the rationalization process under which reasoning according to Battigalli's (1996) best rationalization principle yields the same outcome irrespective of whether players observe their opponents' choices or not. To this respect we find that the obtention of the backward induction outcome crucially depends on tight higher-order restrictions on beliefs about opponents' perfect information. The analysis provides a new framework for the study of uncertainty about information structures and generalizes the work by Battigalli and Siniscalchi (2002) in this direction.

1. Introduction

1.1. Uncertainty about the information structure: an example

Assumptions regarding common knowledge of the information structure of an economic model can significantly impact predictions. Take for instance the sequential Battle of Sexes with perfect information represented in Fig. 1. Two players, Alexei Ivanovich (A) and Polina Alexandrovna (P) choose first and second respectively between actions *left* and *right*, and obtain utility depending on each history of actions according to the numbers depicted at the bottom of the tree in the picture. By information structure we refer to whether or not Polina observes Alexei's earlier choice before she chooses, which she does in this case of perfect information. The game is played just once, so punishment and reinforcement issues are assumed to be negligible. This description is common knowledge among the players, and we further assume that both of them are rational, and that Alexei believes Polina to be rational. It then seems reasonable to predict that players' choices will lead to the unique backward induction outcome: (2, 1); since Polina is rational and observes Alexei's choice, she will mimic it regardless of whether it is *left* or *right*. Alexei believes all the above, so since he himself is rational too, he will move *left*.

Now consider a commonly known imperfect information situation (Fig. 2): consider the alternative information structure according to which, when her turn arrives, Polina will not have observed Alexei's earlier move. Thus, Polina is uncertain of the outcome her choice will induce. Even if it is additionally assumed that Polina believes both that Alexei is rational and that Alexei believes she is rational, it is easy to see that the above argument justifying outcome (2, 1) finds no defense this time; and that indeed, depending on reciprocal beliefs concerning opponents' choices, every outcome is consistent with rationality and with any assumption about iterated mutual beliefs about rationality.

Consider finally an imperfect information case such as the one represented in Fig. 2, with the following variation: Alexei believes himself to be in a situation like the one in Fig. 1; and Polina believes that Alexei believes himself to be in that situation of perfect information. That is, the information structure of the game is not commonly known this time and, in fact, Alexei happens to be deluded about it. When it is her turn to choose, despite not having observed Alexei's earlier move, Polina can infer that since Alexei believes himself to be in a situation with perfect information, he also believes *left* to be followed by *left* and *right* by *right*, and will therefore choose *left*. Hence, despite not observing Alexei's earlier move, Polina believes that Alexei has chosen left and consequently she chooses *left*.

As the example above illustrates, assumptions regarding common knowledge of the information structure of an economic model

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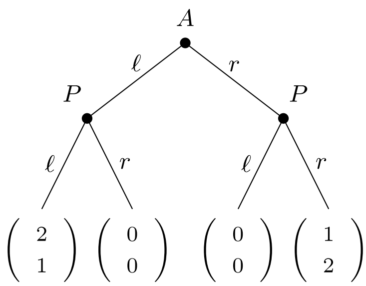

Fig. 1. A game with perfect information.

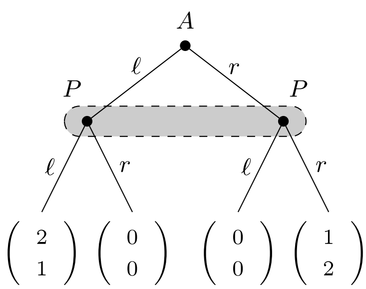

Fig. 2. A game w/o perfect information.

can significantly impact predictions. In order to determine strategic behavior it does not suffice to specify players' ability to observe each others' past choices: careful modeling of the beliefs that players hold about the information structure itself is required too. Consequently, establishing the distinction and exploring the differences in strategic implications between "perfect information" and "common knowledge of perfect information", which refer not only to the way that information flows but also to players' higher-order beliefs about that flow, becomes an interesting issue from a game theoretical perspective. In particular, as the comparison between the first and last situations in the example above suggests, this language enables the class of games for which the backward induction outcome can be considered as a reasonable prediction to be extended to the more general setting of contexts with not necessarily perfect information.

1.2. Information structures and the backward induction outcome: beyond common knowledge of perfect information

Literature on extensive-form games typically assumes that how information sets are distributed along the given game tree in such games is commonly known. This feature can be understood as the information structure of the game being part of the objective rules of the game. However, since information sets describe players' ability to observe, interpret and remember opponents' past behavior, they often depend more on players' personal cognitive abilities than on the rules of the game itself. Thus, since personal cognitive abilities are usually uncertain, it is natural to wonder how predictions in extensive-form games are affected by players facing incomplete information regarding the information structure.

The present paper takes its point of departure from the traditional approach of considering the information structure of an extensive-form game as commonly known, and determines the epistemic assumptions under which the backward induction outcome of the extensive-form is obtained under arbitrary information structures.[1] To that end, we introduce uncertainty about what we call the *information structure* of the extensive-form game. By information structure we refer to how each player's set of histories (i.e. the histories in which it is the player's turn to make a choice) is partitioned into information sets. The information structure can be regarded as players' ability to observe others' past choices, so the uncertainty that we introduce can be read as lack of certainty about whether or not each player is able to observe or remember her opponents' past choices (prior to her turn to choose). To perform our analysis, we first introduce a formal framework that enables incomplete information regarding the information structure of an extensive-form game to be accounted for. In particular, the fact that we allow for each player to face uncertainty about her own information structure means that the traditional notion of information set needs to be broadened to carefully capture the minimum information held by each player whenever it is her turn to make a choice. Next, we present an epistemic framework based on a special kind of conditional belief hierarchies *à la* (Battigalli and Siniscalchi, 1999) (the extensive-form version of Brandenburger and Dekel's (1993) construction of universal type space) that account for uncertainty about information structures. Following this approach, we prove in Theorem 1 that if: (i) players are rational and (ii) there is common strong belief in the event that opponents are rational, have perfect information and strongly believe in their own perfect information, then the backward induction outcome is obtained. Note that we do not assume perfect information: it could be the case that a player does not observe any of her opponents' past choices; still, our common strong belief assumptions enable her to infer what these choices were. Furthermore, we do not impose constraints on each player's beliefs about her own information structure: every player is assumed to believe that her opponents' have perfect information and strongly believe in their own perfect information, but may hold any arbitrary beliefs about her own ability to observe future choices. Still, the obtention of the backward induction outcome crucially hinges on tight assumptions on higher-order beliefs about opponents' perfect information; this illustrates how strong the assumptions on beliefs must be in order for uncertain information structures to not play a role.

The ability of agents involved in some interaction context to obverse each others' choice is often obvious. It might be obvious that there is perfect information: anti-theft devices in a store tell the owner whether a potential thief decided to steal or not. Alternatively, it might be obvious that there is no perfect information: a seller offers a product whose quality he can choose to a buyer; the latter may not necessarily appreciate the quality of the product prior to purchase. This distinction leads to the canonical classification of extensive-form games into those with perfect information and those with imperfect information, in which it is common knowledge that there is perfect and imperfect information, respectively. It turns out that this apparent dichotomy is non-exhaustive, and it is possible to think of situations in which the presence or absence of perfect information is not that obvious: in the first example above the anti-theft device could just be a cheap fake put there to fool potential thieves, while in the second, the buyer might be an expert on the product who is perfectly able to tell the quality of the option offered. Thus, the expected flow of information is sensitive to many aspects surrounding the context of interaction, and it is not clear why agents should not just agree, but *commonly* agree in their appreciation of these aspects and their influence. It is not the aim of this paper to propose a heuristic mechanism that endogenizes the rising of different beliefs about the information structure but rather to point out the possibility of that structure being uncertain, to highlight the relevance of such uncertainty, and to provide conditions in which the assumption of

[1] Thus, we follow the approach by Di Tillio et al. (2014), according to which certain characteristics typically involved in the description of a game, should be treated as individual epistemic attributes of players, and hence, be captured by the usual notion of type.