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First price package auction with many traders☆

Yasuhiro Shirata

Department of Economics, Otaru University of Commerce, 3-5-21 Midori, Otaru, Hokkaido, 047-8501, Japan

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ABSTRACT

This paper studies a first price package auction in which *multiple sellers* participate in addition to multiple buyers. We generalize the notion of the profit-target strategy which is first introduced as a truthful strategy in a first price package auction with a single seller by Bernheim and Whinston (1986b). We then show that the set of equilibrium payoffs in profit-target strategies is equal to the bidder-optimal core, and is also equal to the set of coalition-proof Nash equilibria. Using this result, we find that any equilibrium payoff vector is weakly Pareto-dominated by the VCG payoff vector for buyers, and that the Walrasian competitive equilibrium payoff vector is weakly Pareto-dominated by some equilibrium payoff vector for buyers, even if goods are substitutes. This contrasts with the first price package auction with a single seller, in which it is shown that if goods are substitutes, then those three outcomes are payoff-equivalent.

1. Introduction

A package auction is a selling mechanism where each buyer bids on bundles of multiple items, or packages. The theory of the package auction recently plays an important role in the real economy. The U.S. and the U.K. governments, for example, sell their bundles of spectrum bands under the guidance of auction theorists (see e.g. Cramton et al., 2006).

In their seminal paper, Bernheim and Whinston (1986b) first analyze a static first price package auction in which only one seller participates. They show that there exist equilibria where each bidder is truth-telling, and the corresponding equilibrium payoffs are in the bidder-optimal frontier of the core (bidder-optimal core). Ausubel and Milgrom (2002) show that the bidder-optimal core outcome is implemented by their dynamic ascending proxy package auction with a single seller.

A relation between this package auction theory and the standard competitive market theory has investigated recently. In an auction with a single seller, Bikhchandani and Ostroy (2002) show that the Vickrey–Clarke–Groves (VCG) outcome is supported by the minimal Walrasian non-linear pricing if and only if all buyers are substitutes, and Ausubel and Milgrom (2002) show that the VCG outcome is equal to the equilibrium outcome of their package auction if the social welfare function is bidder-submodular. Those findings imply that the package auction outcome is equal to the VCG outcome, and is supported by the Walrasian pricing when goods are substitutes. The package auction mechanism provides a strategic foundation of the Walrasian equilibrium.

We, in this paper, investigate a generalized package auction, in which *multiple sellers* participate, in addition to multiple buyers. Our first price package auction with multiple sellers is a decentralized trading mechanism. Each buyer first bids his payment schedule for each seller separately, and then each seller independently sells her endowments. We solve this mechanism by generalizing the study by Bernheim and Whinston (1986b). In the first price package auction with a single seller, they introduce the notion of a Nash equilibrium in *profit-target strategies*[1] (profit-target NE), in which each buyer bids truthfully, and show that the set of profit-target NE payoff vectors is equal to the bidder-optimal core. We show that their result still holds true in the first price package auction with multiple sellers. That is, by generalizing their notion of the profit-target NE, we show that the set of profit-target NE payoff vectors is equal to the bidder-optimal core (Proposition 3). Furthermore, they find that any profit-target NE is coalition-proof. We also show that this result still holds true in the auction with multiple sellers (Proposition 4),

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E-mail address: yasuhiro.shirata@gmail.com.

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[1] The profit-target strategy is originally called a truthful strategy by Bernheim and Whinston (1986b). This paper follows the terminology introduced by Milgrom (2004).

Applying this generalization, we next examine the above relations to other trading mechanisms. As mentioned, the literature has shown that in the auction with a single seller, the payoff-equivalence holds among the profit-target NE, the Walrasian equilibrium, and the VCG outcome if goods are substitutes. However, we argue that this payoff-equivalence fails in the auction with multiple sellers even if goods are substitutes. We find a counter example which demonstrates that the profit-target NE payoff is strictly less than the VCG payoff for some buyer (Observation 2), and another counter example which demonstrates that the Walrasian equilibrium payoff is strictly less than any profit-target NE payoff for some buyer if pricing is anonymous and linear (Observation 3).

Therefore, even if goods are substitutes for all buyers, we find that in our package auction with multiple sellers, any profit-target NE outcome is in the bidder-optimal core, in which each buyer earns a payoff less than his VCG payoff and greater than his Walrasian equilibrium payoff. Those our new findings suggest that the implication of the auction with a single seller might be limited. Once we generalize it to the auction with multiple sellers and multiple buyers, the package auction mechanism would no longer provide a strategic foundation of the Walrasian equilibrium even if goods are substitutes.

1.1. Relations to the literature

We, in this section, further discuss relations to the existing literature. First, consider a market in which only a single seller participates. When the seller auctions a single indivisible good, it is well-known that in the second price (or VCG) auction, which is strategically equivalent to the English auction, the equilibrium price in dominant strategies is equal to the minimal Walrasian price. However, in auctions with multiple goods, the equivalence between the VCG auction price and the Walrasian price fails. When the seller sells multiple homogeneous goods, Ausubel (2004) constructs an ascending auction where the equilibrium outcome is equal to the VCG outcome, but is not supported by any anonymous and linear Walrasian pricing.

Allowing non-linear and non-anonymous pricing recovers the equivalence. Even when the seller sells multiple heterogeneous goods, Bikhchandani and Ostroy (2002) show the VCG outcome is supported by the minimal non-linear and non-anonymous Walrasian pricing if and only if buyers are substitutes. de Vries et al. (2007) show that there exists an ascending auction implementing the VCG outcome if the social welfare function is bidder-submodular, but no ascending auctions implement it otherwise. Using discounts at the end of the auction, Ausubel (2006) shows that his ascending auction implements the VCG outcome if valuations are concave, and Mishra and Parkes (2007) generalize this result to arbitrary valuations including non-substitutes valuations. To summarize, when there is only a single seller in a market, the equivalence holds among the ascending auction outcome, the VCG outcome, and the Walrasian outcome. The VCG outcome is achieved by the ascending auction, and is supported by the Walrasian pricing when buyers are substitutes and non-linear and non-anonymous pricing is available.

Next, consider a market in which multiple sellers participate. Then, there are two classes of auction mechanisms. One is a class of centralized mechanisms including the VCG mechanism and the Double auction. Those require the existence of a unique auctioneer or a unique market maker who is able to collect all messages from all buyers and all sellers, compute an array of trades and prices, and impose them. If pricing is anonymous and linear, it is shown that the Double auction approximately implements the Walrasian equilibrium (e.g. Cripps and Swinkels, 2006), but it is also known that the VCG outcome is supported by no Walrasian pricing (Bikhchandani and Ostroy, 2002).

In contrast, we investigate a class of *decentralized* auction mechanisms, in the sense that each buyer's message is a collection of separate messages sent to different sellers—one for each seller, and all actions of a particular seller and her final allocation is independent of messages that the buyers send to other sellers. Our motivations are given as follows. First, implementing the centralized mechanism would be difficult because of computational complexity in a large market consisting of multi-buyers and multi-sellers (Cramton et al., 2006). Second, the centralized mechanism may yield low revenue. Matsushima (2012) shows that central market maker earns negative revenue in the VCG mechanism if and only if the marginal core is non-empty. Third, emergence of market makers is often prohibited by the government from the view of antitrust law.

A decentralized auction with multiple sellers, in addition to multiple buyers, is first studied by Peters and Severinov (2006). They assume that valuations are private information, but all buyers have single-unit demands and all sellers have single-unit supplies. It is shown that there exists a symmetric perfect Bayesian equilibrium which results in the VCG outcome and is supported by the minimal Walrasian pricing. Anwar et al. (2006) support their prediction by testing data from competing auctions in eBay. We study a general package auction in which all buyers have multi-object demands and all sellers have multi-object supplies by assuming complete information, and find that, in contrast to their studies, the package auction outcome is not equal to the VCG outcome and not supported by any anonymous and linear Walrasian pricing in Sections 4.1 and 4.2.

Finally, we discuss a relation to general games with multiple principals and multiple agents. A first price package auction with a single seller is a special class of common agency games with multiple principals and a single agent developed by Bernheim and Whinston (1986a). Prat and Rustichini (2003) extends common agency games to general games with many principals and many agents, and show that the outcome is efficient in any equilibrium in weakly truthful strategies, which is an extension of truthful strategies defined by Bernheim and Whinston (1986b). Since our first price package auction with multiple sellers is a special class of their games and we focus on truthful strategies, the auction outcome is also efficient (Proposition 2). Moreover, by restricting games to the class of first price package auctions, we obtain further characterization. Any profit-target NE is not only efficient, but also bidder-optimal (Proposition 3) and coalition-proof (Proposition 4).

The rest of the paper is organized as follows. Section 2 introduces a decentralized two-sided market in which multiple sellers and multiple buyers participate, and models our first price package auction. Section 3 characterizes equilibrium outcomes. We show that the set of profit-target NE payoffs is equal to the bidder-optimal core, and equal to the set of coalition-proof NE payoffs. Sections 4.1 and 4.2 discuss relations to the VCG mechanisms and to the Walrasian equilibrium, respectively. Section 5 concludes with several remarks.

2. Preliminaries

2.1. A pure exchange two-sided market

In a pure exchange two-sided market E, there are b buyers indexed by $i = 1, \ldots, b$ and s sellers indexed by $j = 1, \ldots, s$. Let I and J be the set of buyers and the set of sellers, respectively. They trade N types of goods indexed by $n = 1, \ldots, N$. Each good n is perfectly divisible. We denote a package or a bundle of goods by $z \in \mathbb{R}^N_+$.