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Optimum cost design of controlled cable stayed footbridges

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ABSTRACT

Traditionally, structures and control devices are designed separately. Here an integrated approach is proposed and applied to find the least cost solution of a passive and active cable stayed footbridge. The optimization process reduces simultaneously cost, stress, acceleration and displacement. By using an entropy-based procedure a Pareto solution is obtained by unconstrained scalar function minimization and an efficient polynomial convergence algorithm is employed. The designed controller is compared with an active *linear quadratic regulator* (LQR). Numerical simulations show that both passive and active optimum designs are efficient, with different geometry, mass distribution and cost (22% higher in the passive design).

1. Introduction

There is a growing interest in applying control to protect civil engineering structures. The use of control devices in cable stayed bridges has been studied in order to mitigate the effects of wind and seismic events. The flexibility of pedestrian cable stayed bridges results in an amplified response when subjected to dynamic loads. Reducing this problem is of vital importance for safety and serviceability [1]. Many control algorithms and devices have been studied to protect structures against seismic events. Semi-active systems are also an attractive alternative for vibration reduction due to its mechanical simplicity, low power requirements and large control force capacity [20]. Benchmark structural control problems for cable-stayed bridges have allowed researchers to compare the efficiency of control algorithms and devices [5]. *Linear quadratic regulator* control design (LQR) has been shown efficient in reducing the dynamic response of the structure [4,11]. Other control algorithms such as the H_∞ [22] and the optimal polynomial controller [10] have also shown to be effective for structural control. Magaña et al. [13] proposed an innovative control scheme which uses active cables in the bridge. The control is decentralized, meaning that each active cable uses only local information (displacement and velocity at anchorage point) to determine actuation.

The optimization of cable-stayed bridges can be stated as the minimization of structural cost or volume and the maximum stresses, displacements and deflections throughout the structure [9].

Negrão and Simões [18] proposed a method to optimally design three dimensional cable-stayed bridges. Erection stages and seismic events were considered in the optimization by both a spectral and time-history-based procedure. Deterministic optimization was enhanced by reliability performance and formulated within the probabilistic framework of reliability based design [19].

The use of control allows the engineer to use different structural systems, this way an integrated structural/control design is viewed as a necessary evolution.

Messac [15] implemented physical programming to the optimum control of a spacecraft example. The design variables were parameters which determine geometric properties of the structure such as mass, damping and stiffness distribution.

Tzan and Pantelides [21] presented a methodology to optimally design an active frame subject to seismic excitation, the objective being minimize the structural volume with constraints of story drift and stresses. Khot [12] proposed an approach to optimally design integrated space systems using multi-objective optimization with goals of minimum volume, control force and time to suppress oscillations. In 2008 Cimellaro et al. [2] illustrated a two-stage optimization procedure for designing active steel frames. In 2009 Cimellaro et al. [3] extended their work to account for inelastic structures. The technique proved efficient in determining the optimal control/structural system. Ferreira and Simões [8] applied an integrated optimum control design to a three span cable-stayed subject to seismic events.

Steel footbridges are usually very flexible structures this, added to their low inherent damping, amplifies their response under dynamic loading. SETRA [16] published a comprehensive guide for the design of such structures highlighting the importance of their

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dynamic properties. In this context the authors are proposing here the integrated design problem of a footbridge with an active tendon and compare its efficiency with a passive design. The steel footbridge presented is intended to guarantee serviceability along running events such as a marathon. Only vertical vibrations are accounted so a two dimensional model is used. The objective is to find two different optimum solutions associated with passive and active bridges. The current European regulation, EC0 [6] and EC1 Part 1–2 [7] are employed.

The design technique uses a multi-objective optimization format with goal of minimum cost, stresses, deflections, accelerations and a Pareto solution is sought. An entropy-based methodology is used to determine the minimax solution by the minimization of a convex scalar function.

The controller designed using the optimization algorithm is compared to a LQR formulation.

2. Optimization strategy

2.1. Minimax objective function

The objective of this work is to find the least cost solution that guarantees safety and serviceability. The problem has one objective function, which is the construction cost. Considering DC_i as the ith design criteria, the optimization problem can be formulated as:

Minimize cost

$$St\, DC_i \leq DC_{imax} \text{ or } \frac{DC_i}{DC_{imax}} - 1 \leq 0 \tag{1}$$

The goal is to find the design variables (DV) that define the optimum cost structure. There are a huge number of restrictions involved in the problem, arising from the static and dynamic time-history analysis. Finding the active constraints in each iteration reveals to be a very time consuming method in such cases [8,18]. Instead the problem is transformed in an equivalent minimax optimization problem (Eq. (2)).

$$\min_{\overline{DV}} \max_i \left\{ \frac{DC_i}{DC_{imax}} - 1, \frac{Cost}{C_{\text{ref}}} - 1 \right\} \tag{2}$$

In minimization problems, one solution vector is said to be Pareto optimal if no other feasible vector decrease one objective function without increasing at least another one. The optimum vector usually exists in practical terms and is not unique [17]. The minimax problem (Eq. (2)) is equivalent to the optimization problem (Eq. (1)) as long as the reference cost (C_{ref}) is continuously updated throughout the optimization process. The minimax problem is discontinuous and non-differentiable, properties that difficult its numerical solution by direct means.

In each iteration the cost and all the DC are determined along with their sensitivity to the DV. The sensitivity analysis was performed using the finite difference method, this way no direct programming of stiffness and mass matrix partial derivates were needed.

After the cost, the DC and respective sensitivities have been determined, the solution procedure adopted was to cast the objectives according to the minimum entropy principle. The problem was formulated as a Kreisselmeyer–Stainhauser scalar function F [17]. This form leads to a convex approximation of the objective and constraint boundaries (Eq. (3)). Accuracy increases with ρ.

$$Minimize\ F = \frac{1}{\rho} ln \left[\sum_{i=1}^{N_{\text{DC}}} e^{\rho \left[DC^i(\bar{v}) + \sum_{j=1}^{N_{DV}} \left(\frac{\partial DC^i(\bar{v})}{\partial v_j} \times v_j \right) - 1 \right]} \right] \tag{3}$$

Legend:

NDC Number of design criteria.
NDV Number of design variables.
DCi(v) Design criteria number *i*.
vj Design variable number *j*.
Δ*vj* Design variable number *j* increment.

The strategy adopted was to solve an iterative sequence of explicit approximation problems. Solving for particular numerical values of the objectives forms one iteration of the complete solution of problem. The solution vector of such iteration represents a new design which needs to be analyzed and checked for safety and serviceability. Iterations continue until changes in the design variables become small. The problem is solved by the steepest descent algorithm as it proved to converge faster (Eq. (4)).

$$\bar{v}_{i+1} = \bar{v}_i - \lambda \times \nabla F_i \tag{4}$$

where ∇F_i stands for the gradient of F. This way the optimization problem with multiple design variables is transformed into one design variable (λ) optimization.

3. Time history analysis and controllers

3.1. Time history analysis

The direct analytical integration method was considered in the step by step procedure, due to its drastic reduction of computational effort. After computing the mass, damping and stiffness matrixes and the force vector (***M***, ***C***, ***K*** and ***f*** respectively), the evaluation of the structural response (***u***) needs solving the dynamic equilibrium equation (Eq. (5)).

$$M\ddot{u} + C\dot{u} + Ku = f \tag{5}$$

The dynamic equation has an analytical solution [14]. Considering the state space vector ***x*** defined as (Eq. (6)):

$$x = \begin{bmatrix} u \\ \dot{u} \end{bmatrix} \tag{6}$$

The 2nd order equation (Eq. (5)) is replaced by a 1st order (Eq. (7)).

$$\dot{x} = Ax + Bf \tag{7}$$

The matrixes ***A*** and ***B*** are defined by Eqs. (8) and (9) respectively.

$$A = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix} \tag{8}$$

$$B = \begin{bmatrix} 0 \\ -M^{-1} \end{bmatrix} \tag{9}$$

Eq. (10) gives the analytical solution to Eq. (7) for a given time step Δt.

$$x(t + \Delta t) = e^{\Delta t A} x(t) + \int_t^{t+\Delta t} e^{[(t+t)-\zeta]A} Bf(\zeta) d\zeta \tag{10}$$

Assuming the forces in the structure vary linearly, dynamic time history analysis can be done using the following equation (Eq. (11)).

$$x(t + \Delta t) = K_b x(t) + K_f Bf(t) + K_{\Delta f} B[f(t + \Delta t) - f(t)] \tag{11}$$

Eq. (11) uses information on the current time step, and determines the state in the next time step. The dynamic matrixes $\boldsymbol{K_b}$, $\boldsymbol{K_f}$ and $\boldsymbol{K_{\Delta f}}$ are determined using Eqs. (12)–(14). Where ***I*** represents the identity matrix.

$$K_b = e^{\Delta t A} \tag{12}$$