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Noisy signaling in discrete time☆

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ABSTRACT

This paper characterizes the equilibrium set of a dynamic noisy-signaling model in discrete time. A seller privately knows the quality of her asset. She can exert a costly effort to generate stochastic returns. Buyers stochastically arrive over time and, after observing the history of returns, they make price offers. In our model, the equilibrium behavior of the buyers is discontinuous: they only make acceptable (high) offers if the posterior about the quality is above a given threshold. As a result, the recursive nature of the model replicates the discontinuity, giving the equilibrium continuation payoff a complex self-replicating structure that may take the form of a devil's staircase.

1. Introduction

This paper characterizes the set of perfect Bayesian equilibria of a dynamic noisy-signaling model in discrete time. We show how the recursive nature of dynamic signaling shapes the equilibrium objects, such as payoffs and effort choices. Our analysis reveals the challenges that endogenizing the informativeness of stochastic signals poses in discretized dynamic models, where local discontinuities may have global effects on the equilibrium behavior.

The analysis focuses on the following dynamic trade model. A seller wants to sell an asset, which can have a low or a high underlying quality, also referred to as type. Only the seller observes the quality of her asset, and she can exert an unobservable effort that generates observable noisy returns. The cost of signaling is type-dependent, and absent signaling motives, the efficient effort differs across types. Short-lived buyers, who stochastically arrive over time, observe the history of returns and make offers to the seller. If the seller accepts an offer, the asset is sold, and the game ends. Otherwise, the seller continues managing her asset until the arrival of the next buyer.

We assume that there are no gains from trade for the low-quality asset, and as a result, buyers never make acceptable offers intended only for the low-quality seller. Also, due to Diamond's paradox, buyers use their local monopoly power to extract all surplus from the high-quality seller in all equilibria. As a result, the different types of the seller pool on the acceptance decision and the high-quality seller has strict incentives to manage her asset optimally. In equilibrium, separation comes from different effort choices across the types of the seller.

If the cost of effort is high enough, our model features a unique equilibrium outcome. In most periods, the low-quality seller randomizes between exerting her efficient (low) managerial effort versus masquerading her type by exerting a suboptimal (high) effort. Intuitively, if the low-quality seller is supposed to exert a low effort, the signal becomes very informative. In this case, high returns would convince future buyers that the quality of the asset is high, so the seller has incentives to undertake a cost-inefficient (but revenue-generating) effort in order to increase her expected revenue from selling the asset. The reverse is true if she was supposed to exert a high signaling effort: since the signal would be uninformative, the incentive to exert a high effort would be very low. The effort exerted by the low-quality seller in mimicking the high-quality seller is high for intermediate posteriors about the quality, as it is there where beliefs are updated fast. Alternatively, if buyers believe that, with a high probability, the quality of the asset is high, the low-quality seller exerts a low effort for some periods, hoping to sell the asset at a high price before buyers become pessimistic.

We explicitly construct the "equilibrium continuation payoff set", which is the graph of the correspondence that maps each prior about the quality of the asset being high to the corresponding equilibrium continuation payoffs for the seller of the low-quality asset. Since buyers make acceptable offers only when the posterior is above a given threshold, the continuation payoff correspondence is not lower-hemicontinuous at this threshold. The effect of this

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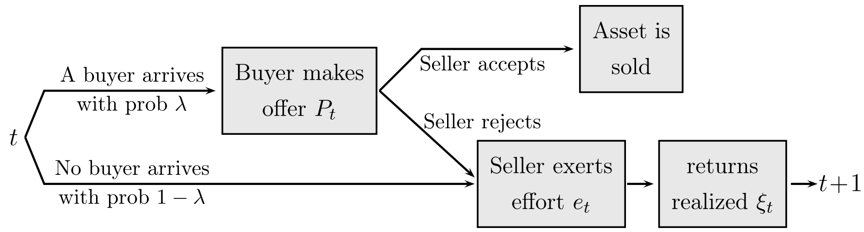

Fig. 1. Timing of the model.

discontinuity replicates itself due to the recursive structure of the continuation values, giving the equilibrium continuation set a step structure. In particular, it may take the form of a devil's staircase (or Cantor function), that is, a non-constant continuous function that is flat almost everywhere. As a result, if the prior (i.e., composition of the market) is endogenized by introducing an entry fee, it is discontinuous in a dense set with respect to the entry fee. Furthermore, in this case, the expected equilibrium managerial effort features an infinite number of peaks and valleys.

When the cost of effort is low, there are equilibria where, even if no buyer arrives for a long time, the type of the seller is not revealed. In this case, an offer from a buyer is high only if recent realizations of the returns are also high, which incentivizes the low-quality seller to exert a high effort. So, pooling is sustained in equilibrium by rewarding high managerial effort with a high probability of receiving a high offer.

Applications. There are a number of economically relevant situations where an agent has inside information not only about a state of the world (or type), but she also can undertake actions which cannot be observed by outsiders, and which stochastically affect some signals. A prominent example is the sale of an asset or a patented business idea by an entrepreneur. In this case, potential buyers may only observe successful prototypes, prizes or patents. Similarly, employers may learn about the productivity of a potential employee only through the observation of signals like her grades during her high education, her successful scholarship applications or her student prizes. Finally, politicians may signal their skills through successfully passing their proposals in the parliament or by wining local elections. In these examples, some outcomes are observable or incentive-compatible to disclose, while effort or other costs may be difficult to observe or credibly report. Also, even though the accumulation of signals over time generates a rich signal about the private persistent information, each individual signal is heavily discretized.

The organization of the rest of the paper is as follows. In Section 2, we set our base model and the main results of the paper. Section 3 discusses the literature and concludes. Appendix provides the proofs of all the results and a continuous-time limit of our base model.

2. Basic model

2.1. Setting

Time is discrete, $t = 0, 1, 2 \ldots$. There is a (female) seller who wants to sell an asset. The asset is either of low quality ($\theta = L$) or of high quality ($\theta = H$). The quality of the asset, also referred to as type, is known to the seller. The seller discounts future payoffs at a discount factor $\delta \in [0, 1)$.

As long as the asset has not been sold, in every period t, the seller decides on the effort $e_t \in \{0, 1\}$ put into managing the asset. We assume that high effort can seem low, but low effort cannot seem high. More precisely, if the effort exerted at t is e_t, the asset generates returns at t equal to $\pi_G \equiv \pi > 0$ with probability $\nu\, e_t$ and $\pi_B \equiv 0$ with probability $1 - \nu\, e_t$, for some $\nu \in (0, 1)$. We use $\xi_t \in \{B, G\}$ to denote the realization of the returns at time t, where B denotes that returns were low at t ("bad" signal), while G denotes that returns were high at t ("good" signal).

The cost of providing effort e is type-dependent and normalized to $c_\theta\, \nu\, e$ for $\theta \in \{L, H\}$. Note that c_θ can be interpreted as "the cost per unit of the probability of generating high returns". We assume that $c_H < \pi < c_L$. This implies that, absent signaling motives, high effort is optimal for the H-seller, but not for the L-seller. We define $\overline{V}_L \equiv 0$ and $\overline{V}_H \equiv \nu\, \frac{\pi - c_H}{1-\delta}$ as the autarchy values of the L-seller and the H-seller, respectively.

There is an infinite pool of homogeneous short-lived (male) buyers. In every period, there is a probability $1-\lambda$ that no buyer arrives, and a probability $\lambda \in [0, 1]$ that (exactly) one buyer arrives. Buyers value an asset of quality $\theta \in \{L, H\}$ at U_θ, with $U_L < U_H$, and share a prior $p_0 \in (0, 1)$ on the asset's quality being H. We assume $U_H > \overline{V}_H$ (gains from trade for the H-asset) and $U_L < \overline{V}_L$ (no gains from trade for the L-asset).[1] If a buyer arrives at t, he (only) observes the previous history of returns. Then, he makes a (private) take-it-or-leave-it offer to the seller $P_t \in \mathbb{R}$. If the seller accepts the offer, the asset is sold and the game ends. Otherwise, the game continues. The timing of the game is schematically displayed in Fig. 1.

We focus our analysis on the case where exerting high effort is not too cheap for the L-seller or, alternatively, when the arrival probability of buyers is small enough. As we will see, this will rule out pooling as equilibrium behavior.

Assumption 1. $c_L - \pi > \delta\, \lambda\, \overline{V}_H$, that is, high effort is costly for the L-seller.

We presume Assumption 1 in all sections except for Section 2.6, where we relax this assumption and provide some intuition on why it is important to characterize equilibrium behavior.

Histories and payoffs

A (unterminated) *public history* is an element of $\mathcal{H} \equiv \cup_{t=0}^{\infty}\{B, G\}^t$ and it encodes the returns realized in the past. A (unterminated) *private history* of the θ-seller, for $\theta \in \{L, H\}$, is an element of $\tilde{\mathcal{H}} \equiv \cup_{t=0}^{\infty}\big(\{B, G\} \times \{0, 1\} \times (\{-\infty\} \cup \mathbb{R})\big)^t$, that is, a public history plus the effort choices by the seller and the offers made by the buyers, where an offer equal to $-\infty$ at time t corresponds to no buyer arriving in this period. A *terminated private history* $(\tilde{h}^t, P) \in \tilde{\mathcal{H}} \times \mathbb{R}$ is composed of a private history and the offer accepted after it (at time t).

A strategy of a buyer who arrives at time t with public history h^t is given by a distribution over the price offers $\tilde{P}(h^t) \in \Delta(\mathbb{R})$. A strategy by the θ-seller, for $\theta \in \{L, H\}$, is an acceptance decision rule $\beta_\theta : \tilde{\mathcal{H}} \times \mathbb{R} \to [0, 1]$, where $\beta_\theta(\tilde{h}^t, P_t)$ is the probability of accepting an offer P_t at history $\tilde{h}^t$, and an effort choice $\alpha_\theta : \tilde{\mathcal{H}} \times (\mathbb{R} \cup \{-\infty\}) \to [0, \nu]$, where α_θ/ν is the probability of choosing effort

[1] Since $\overline{V}_L = 0$, we have $U_L < 0$, which may seem counter-intuitive, especially given the usual assumption of free disposal of the asset. Nevertheless, notice that $\pi_B = 0$ is just a normalization, so in general $\overline{V}_L = \frac{\pi_B}{1-\delta}$. Furthermore, transaction (legal/taxes) costs may reduce the buyers' valuation of the asset.