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A simple macroeconomic model with extreme financial frictions☆

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ABSTRACT

We develop a simple macroeconomic model with extreme financial frictions (no credit markets) and show that poverty traps can emerge even in the absence of leverage. In our model, farmers produce fruit by renting land from landlords. Crops are exposed to aggregate shocks (weather risk). To guarantee themselves a positive consumption level even after a bad crop, farmers store fruit as precautionary savings and adjust their scale of activity to the level of these savings. The land that is not rented to farmers is cultivated by landlords, who are less productive. We show that there is a unique Markov competitive equilibrium, in which the rental price of land increases with the level of farmers' savings. A decline in savings, caused by a bad crop, may bring the economy into a "poverty trap", even in the absence of any leverage. Fluctuations of output are caused by productivity shocks and amplified by fluctuations in the level of activity of farmers. The simplicity of our model allows us to study analytically why the long run behavior of the economy may differ markedly from the one predicted by the steady state paradigm. Specifically, we show that when the risk-adjusted productivity of farmers is high and the elasticity of the land supply is low, using the steady state paradigm leads to serious mis-estimations of the long run average state of the economy.

1. Introduction

In the wake of the financial crisis, a need to better understand the sources of financial instability gave rise to a new generation of continuous time general equilibrium models such as Brunnermeier and Sannikov (2014) or He and Krishnamurthy (2012, 2013). One remarkable feature of these models is that they allow to analyze the full macroeconomic dynamics. In particular, Brunnermeier and Sannikov (2014) show that the long run behavior of an economy confronted with aggregate shocks and financial frictions can be in a sharp contrast with the one predicted by the steady state analysis that is typically used in standard macroeconomic models.[1] The main reason for this divergence is the presence of system-generated (endogenous) risk that may drive the equilibrium dynamics far away from the state in which the system would end up in the absence of such risk. Taking into account this feature is critical if one intends to address welfare distortions or policy design.

However, the aforementioned macro models suffer from two major drawbacks: they can only be solved numerically and some of their predictions are at odds with empirical evidence. In this paper, we build on Kiyotaki and Moore (1997) and Brunnermeier and Sannikov (2014), and develop a simple dynamic macro model that lends itself to quasi-closed form solutions and fits better some stylized facts. For example, one of the main drivers of output fluctuations in US data seems to be the changes in the level of activity (number of hours worked) (see e.g. Hodrick and Prescott, 1997 or Ohanian and Raffo, 2012). Moreover, most firms hold sizable buffers of liquid reserves as precautionary savings (see e.g. Bates and Stulz, 2009), and systematic (aggregate) risk is identified by some empirical studies as a key driver of liquidity hoarding

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[1] They illustrate this feature by giving an example of an economy that can be trapped for a long time in recessions.

behaviors (see e.g. Achariya et al., 2013, Gao and Grinstein, 2014). Our model captures these features.

In the spirit of Kiyotaki and Moore (1997) we have two classes of agents, risk-neutral landlords and risk-averse farmers who cultivate the land that they rent from landlords.[2] Land yields a stochastic harvest of fruit ("apples") that can be consumed or stored in reserves. Financial frictions are extreme in the sense that agents cannot access financial markets. In particular, farmers cannot borrow as they do not possess any collateral. Moreover, they cannot insure against harvest shocks. To avoid defaulting on their rent contracts, farmers adjust their scale of activity to the level of their reserves. Equilibrium of supply and demand on the rental market for land determines the dynamics of the rental rate.

The simple structure of our model enables us to expose the nature of the non-linear macrodynamics that arise generically in dynamic macro models with financial frictions à la Brunnermeier and Sannikov (2014). Being able to derive closed form solutions allows us, in particular, to explicitly track down the roots of some remarkable features of these models such as the *paradox of volatility* and *persistence* of exogenous shocks. The latter can manifest itself in our setting as a *poverty trap*, when the economy is stuck for some time in a situation characterized by low levels of savings and low rental rates. All these phenomena arise as the consequences of endogenous (system-generated) risk that emerges because of fluctuations of farmers' risk tolerance, due to shocks to their savings. When these savings are low, the demand for land is also low, driving down the rental rate. At the reduced rental rate, some (less productive) landlords prefer to (inefficiently) cultivate land themselves instead of renting it to farmers, which prevents the immediate recovery to the initial scale of farmers' activity even if an equivalent positive harvest shock occurs.

To better understand the implications of endogenous risk for the long run macrodynamics, we compute the ergodic density function of the rental rate and compare the long-run behavior of the economy in a fully dynamic set-up with the one predicted by the steady state analysis. Knowledge of the ergodic density enables us to compute the *long run average* of all economic variables of interest. We first compute the long run average of the rental rate and compare it to the steady state rental rate. In our model, whether the long run average of the rental rate is well approximated by the steady state value or not depends on two parameters: the risk-adjusted productivity of farmers and the elasticity of the land supply. In particular, we show that the long run average of the rental rate is close to the steady state level when the risk-adjusted productivity of land cultivated by farmers is low and the elasticity of the land supply is high. In this case the impact of endogenous risk on macrodynamics is relatively weak. By contrast, when the risk-adjusted productivity of farmers is high and the price-elasticity of the land supply is low, the steady-state approximation may lead to serious over/underestimations, as it neglects the role of endogenous risk for the system dynamics.

Next, we illustrate the implications of endogenous risk for welfare analysis. We compute the time average of the instantaneous welfare loss, showing that its value can substantially differ from the instantaneous welfare loss evaluated at the steady state. This divergence is particularly pronounced when the elasticity of the land supply is low. We demonstrate that a poor quality of the steady-state approximation may come from two kinds of approximation errors: a "mean" error caused by the difference between the long run average rental rate and the steady state rental rate, and a "higher-order" error, caused by neglecting endogenous risk in the system together with non-linearities of the loss function. This result shows that, in general, approximating economic variables of interests by their steady-state values can be misleading.

From a methodological perspective, our paper belongs to the recent but rapidly growing literature on continuous-time macrofinancial dynamics (see e.g. Brunnermeier and Sannikov, 2014, He and Krishnamurthy, 2012, 2013).[3] This literature seeks to understand the macroeconomic implications of financial frictions, by focusing essentially on the interplay between agents' net worth, asset prices and financing constraints. He and Krishnamurthy (2012, 2013) put emphasis on capital constraints. In their models the interplay of low net worth of more productive agents and binding capital constraints gives rise to endogenous risk, which drives up the premium on risky assets. However, the economy recovers relatively quickly from recessions so that its long run behavior does not substantially differ from the one that would be predicted by the steady state analysis. Brunnermeier and Sannikov (2014) consider a leverage constraint that prevents agents from borrowing more than the market value of their assets. In contrast to He and Krishnamurthy (2012), endogenous risk is permanently present in their economy and may drive its dynamics far away from the steady state. Endogenous risk in their setting generates a negative externality of agents' risk-management decisions made in the presence of financial frictions. In particular, when the share of net worth held by the more productive agents declines, these agents reduce their exposure to aggregate shocks by selling assets to less productive agents. Fire-sales put downward pressure on asset prices, which creates complex feedback loops affecting the macroeconomic dynamics. In contrast to Brunnermeier and Sannikov (2014), our model studies the implications of endogenous risk in a much simpler setting that abstracts from asset sales. Instead, similar to the partial equilibrium model of Isohätälä et al. (2014), we consider a rental market for land in which farmers do not own land but rent it from landlords. This simple setting allows us to derive explicit dynamics, while preserving essential general equilibrium effects that are inherent in the new macroeconomic models of similar style.

The rest of the paper is structured as follows. Section 2 presents the model. Section 3 describes the equilibrium and discusses its main properties. Section 4 studies the long run behavior of the economy. Section 5 compares these findings with the predictions of traditional impulse response analysis. Section 6 reports the results of welfare analysis. Section 7 concludes. All mathematical proofs are gathered in the Appendix.

2. The model

2.1. Technologies and preferences

The economy is populated by two types of agents: risk-neutral landlords and risk-averse farmers. Both types of agents discount the future at the same rate ρ. The total stock of land in the economy is fixed and normalized to 1. All land belongs to landlords and can be uniquely used for growing apple trees. Farmers can rent land from landlords at a *rental rate* q_t which will be endogenously determined in equilibrium.

Farmers are more efficient in cultivating land than landlords. In particular, when cultivated by a representative farmer, one unit of land yields the flow of output (apples)[4]

$$dy_t = adt + \sigma_0 dZ_t, \tag{1}$$

[2] In our model, it would be inefficient for farmers to buy land rather than rent it, because they are risk-averse and landlords are risk neutral. Thus it is optimal to allocate capital risk to the landlords.

[3] For an extended review of this literature, we refer the reader to Isohätälä et al. (2016).

[4] Throughout the model apples are used as the numeraire.

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