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International borrowing without commitment and informational lags: Choice under uncertainty[☆,☆☆]

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ABSTRACT

A series of recent studies in economic growth theory have considered a class of models of international borrowing where, in the absence of a perfect investment commitment, the borrowing constraint depends on the historical performances of the country. Thus, a better level of past economic activity gives a higher reputation, thereby increasing the possibility of accessing the international credit market. This note considers this problem in a stochastic setting based on the volatility of the internal net capital. We study how the optimal consumption level and the maximal expected welfare depend on the combined influence of the trajectory of past economic variables and the volatile environment. In particular, we show how the strength of the *history effect* and the relative weight of the historical performance depend on the degree of risk.

1. Introduction

Modeling constraints on access to the international credit market for small or highly indebted countries is a lively issue at present. A possible approach to address this question was proposed by Boucekkine and Pintus (2012) based on an intuition of Cohen and Sachs (1986). They relaxed the unrealistic assumption of commitment to investment by considering the importance of the "historical course" of the economy. In particular they assumed that, in the impossibility for the debtor country to commit to an investment strategy, the lender bases its decisions on past investments, and thus the past path of the capital stock.

The no-commitment delay between the past capital measure and the current borrowing capacity (and thus the current investment possibilities) is the basis of the *history effect* emphasized by Boucekkine and Pintus (2012), which allows their model to replicate a series of macroeconomic instability behaviors, such as growth break and growth reversal phenomena that are recurrent and well documented (e.g., see Jones and Olken, 2008 or Cuberes and Jerzmanowski, 2009), and to justify their relationship with the process of financial integration.

Boucekkine et al. (2013) (and a companion paper by Boucekkine et al., 2011) introduced explicit preferences and optimal saving decisions into the framework of Boucekkine and Pintus' model, which was originally formulated based on the hypothesis of a fixed exogenous saving rate *à la Solow*. In this manner, they studied the welfare implications of financial globalization in the context of the model. They qualitatively replicated the empirical observations of Kaminsky and Schmukler (2008), thereby suggesting that financial globalization can lead to a short-run consumption (and welfare) drop and a long-run gain. In addition, Boucekkine et al. (2013) emphasized the differential impact of financial integration changing the historical economic path, where countries with the same initial capital stock but different paths achieve highly variable results after integration into the international financial system, thereby further demonstrating the importance of history.

In this note, we propose a stochastic version of the model of Boucekkine et al. (2013). In fact, due to problems of analytical tractability, we employ a constant absolute risk aversion (CARA) utility function whereas Boucekkine et al. (2013) focused on the constant relative risk aversion (CRRA) case. Apart from this (and the new stochastic terms), the two models are identical.

Several determinants of risk need to be considered when a country assesses its borrowing choices. First, there is a series of exogenous factors related to exposure to international credit market volatility: as argued, e.g., by Prasad et al. (2007), at least in the

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early stages, financial integration is associated with significant increases in the volatilities of both output and consumption. Second, there is the volatility associated with domestic shocks, which, as shown by Loayza et al. (2007), has an important role especially in the setting of the small, typically developing, countries with open economies that we consider in the present study. In this study, we focus on this second series of phenomena and specifically on the macroeconomic volatility that affects the structure of production, due, for example, to production specialization (e.g., see Kraay and Ventura, 2007) or social conflicts (Raddatz, 2007). Thus, the volatility is linked to the level of net capital (capital net of foreign debt) in our model (see Section 2 for details).

A version of the model without any informational lag was studied by Boucekkine et al. (2014), whereas we model the absence of commitment to investment *à la* Boucekkine and Pintus (2012). A certain number of model predictions, such as the positive effect of volatility on precautionary saving and then on the long-run growth rate, can be described using the simpler stochastic one-dimensional model of Boucekkine et al. (2014), but studying the interaction between the *history effect* and the risky environment needs to introduce the no-commitment delay. Thus, we omit the questions that can be answered clearly using the simpler set-up of Boucekkine et al. (2014), and we focus on those that can only be studied in the new context: how does the history effect change with the characteristics of the economy and what is the role of volatility? Furthermore, does the relative importance of remote events or more recent facts change in different contexts? In particular, can we observe some "oblivious" processes?

To answer these questions, we first characterize the optimal planner solution. In Section 3, we provide the explicit expression of the optimal consumption in feedback form, we characterize the optimal capital trajectory as the solution to a suitable stochastic equation (Theorem 3.3), and we determine the welfare that corresponds to the optimal consumption/saving policy (see Proposition 3.5). These results allow us to consider the structure of the optimal policy in detail, by decomposing its expression in terms of the contributions of the present net capital and of the past capital history and by emphasizing the different weights of different past periods (see Section 4.1). We prove that the total strength of the history effect is not reduced by the volatile environment. This is an interesting corroboration of the solidity of the history effect. Is spite of this we show that the relative weights of the "old" history terms decrease when the environment is more volatile (or in a situation where individuals are more risk averse), whereas recent events become increasingly important; thus, the volatility promotes an "oblivious" process.

The methodological contribution. Several previous studies used delay differential equations (i.e., functional differential equations where the variable appears in delayed form[1]) to model several economic phenomena, but Boucekkine and Pintus (2012) were probably the first to introduce an economic model driven by a neutral differential equation (NDE). In the NDE case, the "past" of the variable and that of its derivative are included in the equation. NDEs are harder to study than delay differential equations: the typical regularizing properties of delay differential equations are not valid in the NDE case (e.g., see Hale and Verduyn Lunel, 1993) and the asymptotic properties are more difficult to prove. However, because systems driven by delay differential equations are already infinite dimensional, *a fortiori* dealing with NDEs involves working in an infinite-dimensional set-up.

A further advance in terms of technical complexity was considered by Boucekkine et al. (2011, 2013), where they had to deal with an optimal control problem driven by an NDE to study their model. As argued by Kolmanovskii and Myshkis (1999, particularly in Chapter 14), the use of the maximum principle is problematic in the NDE case (indeed most previous studies of the control of NDEs consider robust control and optimal control is very rare). Boucekkine et al. (2011) studied the problem by using the tools of dynamic programming in infinite dimensions. A similar approach was already used for simpler cases of models driven by delay differential equations, see, e.g., Fabbri and Gozzi (2008).

An additional difficulty is considered in the present note. The optimal control problem is now driven by a stochastic NDE (i.e., the state equation (8)), i.e., an NDE with an extra stochastic term. This problem is also approached using dynamic programming in infinite dimensions. Provided that the positivity condition on the net capital trajectory is satisfied, we can write the value function expression explicitly and characterize the explicit solution to the problem in closed-loop form (see Theorem 3.3). To the best of our knowledge, this is the first optimal control problem driven by a stochastic NDE to be solved in the (not only economic) literature.[2] The generalization with respect to the deterministic case is not trivial because the stochastic term in the state equation entails a second order term in the infinite dimensional Hamilton–Jacobi–Bellman equation (by contrast, only the first order Fréchet differential appears in the deterministic case) and a stronger regularity is needed to define the regular solutions. For further details, Appendix provides the mathematical apparatus and the necessary proofs.

Structure of the note. This note is organized as follows. In Section 2, we introduce the model and its main features. Section 3 presents the analytical results, in Section 4 we discuss the results and their implications, in Section 5 we present two generalizations of our approach while Section 6 gives the conclusions of this study. Appendix contains the proofs.

2. The model

We consider a small open economy with an aggregate AK production function, where $K(t)$ is the capital input at time t and A is the level of technology. At each time point, the country can borrow on the international credit market at a fixed and exogenous interest rate r.

We denote by δ the depreciation rate of the capital, $C(t)$ and $D(t)$ are the level of the aggregate consumption and the stock of net foreign debt at time t, respectively, and $N(t)$ is noise (as specified below) that perturbs the economy. We assume that the evolution of the variables satisfies the following equation

$$\dot{K}(t) - \dot{D}(t) = AK(t) - \delta K(t) - rD(t) - C(t) + N(t). \tag{1}$$

Excluding the noise N, this is simply the deterministic budget constraint of the economy described by Boucekkine et al. (2013).

Following Boucekkine and Pintus (2012) and Boucekkine et al. (2013), and in the spirit of Cohen and Sachs (1986), we assume that the borrowing capacity of the country depends on the past performance of the economy and particularly that, for any $t \geq 0$,

$$D(t) = \lambda K(t-\tau), \tag{2}$$

[1] Some examples in various domains are: Asea and Zak (1999) or Bambi (2008) in terms of growth models with time-to-build during production, d'Albis et al. (2012) in modeling the learning-by-doing process, Boucekkine et al. (2005) with a vintage capital model, and Feichtinger et al. (1994) with an advertising model.

[2] By contrast, there have been several economic models in the form of optimal control problems driven by stochastic delay differential equations, e.g., see Gozzi et al. (2009), Federico and Tankov (2015), and Fabbri and Federico (2014). However, as in the deterministic case, they are more tractable than optimal control problems driven by stochastic NDEs due to the absence of the delayed derivative term.

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