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# International transmission of bubble crashes in a two-country overlapping generations model☆

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ABSTRACT

We study the international transmission of bubble crashes by analyzing stationary sunspot equilibria in a two-country overlapping generations exchange economy with stochastic bubbles. We consider two cases of sunspot shocks. In the first case, we assume that only the foreign country receives a sunspot shock, while in the second, we assume that both countries independently receive sunspot shocks. In the first case, a bubble crash in the foreign country is always accompanied by a bubble crash in the home country. In the second case, a bubble crash in the foreign country can have a positive or negative effect on the home bubble. We also show that there exists a unique locally isolated stationary sunspot equilibrium, and that it is locally unstable.



## 1. Introduction

The history of financial markets overflows with episodes of asset bubbles (e.g., Kindleberger and Aliber, 2005). Due to global financial integration over the past few decades, financial markets are now highly interdependent across countries (e.g., Tsutsui and Hirayama, 2010; Ehrmann et al., 2011; Madaleno and Pinho, 2012). As a consequence, the bursting of an asset bubble in one country can have significant impacts on financial markets in other countries. This is what may have happened during the global financial crisis of 2007–2008.

Since this event, the macroeconomic literature on asset bubbles has been growing rapidly. Much of the recent literature considers models of bubbles based on financial frictions and examines the real effects of bubbles; see, e.g., Farhi and Tirole (2012), Martin and Ventura (2012), and Miao and Wang (2012a,b).[1] On the other hand, somewhat surprisingly, very little theoretical work has been done on the international transmission of bubble crashes in highly integrated financial markets.

There have of course been some closely related studies. For example, Ventura (2012) showed that bubbles may comove across countries in an overlapping generations model consisting of multiple countries with different levels of productivity. However, in his model, financial markets in different countries are completely segregated. Tandon and Wang (2003) studied currency substitution in a small open overlapping generations model by analyzing the dynamics of a stochastic bubble, but their analysis was restricted to the deterministic dynamics of the bubble prior to its burst. A recent paper by Martin and Ventura (2015) considers the international transmission of credit bubbles, but in their model the bubbles are affected by a common state variable and assumed to comove.

The purpose of this paper is to analyze the international transmission of bubble crashes in fully integrated financial markets. In other words, we wish to understand the effect of the bursting of a bubble in one country on a bubble in another. For example, if a bubble in one country bursts, then what happens to a bubble in another country when the relevant financial markets are fully integrated? This type of question cannot be answered if the bubbles are assumed to comove at the outset.

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[1] See Miao (2014) for a recent survey. See Kamihigashi (2001, 2008, 2015) and the references therein for discussion on the earlier literature on bubbles.

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For this purpose we construct a two-country version of the overlapping generations exchange economy developed by Weil (1987).[2] In our model the countries, called "home" and "foreign", are perfectly symmetric in terms of fundamentals. There is a unique consumption good worldwide, and each country has an intrinsically useless asset, or a "bubble". The good and asset markets are fully integrated internationally; agents in either country have full access to the good and asset markets in both countries.

In this setting, we consider two cases of sunspot shocks. In the first case, we assume that only the foreign country receives a sunspot shock, which has no direct influence on the fundamentals of the economy. A sunspot shock occurs only once over the infinite horizon, with a constant probability in each period. We assume that the bubble in the foreign country bursts if a sunspot shock occurs, and remains at a constant level otherwise. How does the bubble in the home country react to the bursting of the foreign bubble when the home bubble is not required to react at all? We show that the home bubble inevitably bursts simultaneously in response to the bursting of the foreign bubble.

In the second case, we assume that both countries receive sunspot shocks independently. In each country, a sunspot shock occurs only once over the infinite horizon with a constant probability in each period. But this probability is assumed to differ across countries. As in the previous case, we assume that the foreign bubble bursts if a sunspot shock occurs in the foreign country. Likewise, the home bubble bursts if a sunspot shock occurs in the home country. We show that if the foreign bubble bursts, then the home bubble either bursts simultaneously or jumps to a higher level. Hence, unlike in the previous case, a bubble crash in one country can have a positive or negative effect on the bubble in the other country.

The stationary sunspot equilibrium in which a bubble crash in one country has a positive effect on the bubble in the other country is locally isolated. Any other stationary sunspot equilibrium shown in this paper belongs to a continuum of stationary sunspot equilibria. Analyzing the local dynamics around the locally isolated stationary sunspot equilibrium, we show that this equilibrium is locally unstable. Thus to achieve this stationary sunspot equilibrium, the economy must initially jump to this equilibrium.

As discussed above, this paper builds upon the work of Weil (1987) and is most closely related to the literature on asset bubbles. Another strand of literature related to this paper is that on sunspot equilibria in two-country overlapping generations models initiated by Spear (1989) and Manuelli and Peck (1990). In particular, our model is similar to that of Manuelli and Peck, and shares some properties including a common portfolio across countries. However, this literature mostly focuses on exchange rate fluctuations without explicitly considering bubble crashes; see, e.g., Barnett (1992), Betts and Smith (1997), and Russell (2003). To our knowledge, very little is known as to how a bubble crash in one country affects a bubble in the other country. Focusing on this particular issue, this paper seems to be distinct.

A large body of literature in international finance emphasizes the roles of fundamentals (e.g., Kaminsky and Reinhart, 2000), imperfect or asymmetric information (e.g., Allen and Gale, 2000), and financial constraints (e.g., Devereux and Yetman, 2009) as potential sources of international transmission of financial crises and shocks. This paper complements this literature by showing that the international transmission of bubble crashes is an inevitable consequence of financial integration within a simple framework without introducing any friction or fundamental uncertainty.

The rest of the paper is organized as follows. In Section 2 we review the case of a closed economy and show some preliminary results. In Section 3 we introduce the two-country economy, define equilibria, and show some basic results. In Section 4 we assume that only the foreign country receives a sunspot shock. In Section 5 we assume that both countries receive sunspot shocks. In Section 6 we study the local stability of the unique locally isolated stationary sunspot equilibrium. In Section 7 we provide some concluding remarks and discuss possible extensions. All omitted proofs appear in appendices unless otherwise noted.

## 2. The closed economy

In this section, we consider a closed economy that is essentially the same as the exchange economy of Weil (1987). The results in this section are presented for later reference; we do not claim originality here.

### 2.1. General structure

In each period $t \in \mathbb{Z}_+$, a new generation of homogeneous two-period-lived agents are born. They are called young in the first period of their life, and old in the second period. There is no population growth, and the population of each generation is normalized to one. There is a single consumption good, and each agent is endowed with $e_1 > 0$ units of the good when young, and $e_2 > 0$ units when old. There is also an intrinsically useless asset that agents can buy when young, and sell when old. We regard this asset as a bubble whenever its market price is strictly positive.

Each agent born in period $t \in \mathbb{Z}_+$ solves the following maximization problem:

$$\max_{c_t, x_t, d_{t+1} \geq 0} \quad u(c_t) + \mathbb{E}_t v(d_{t+1}) \tag{2.1}$$

$$\text{s.t.} \quad c_t + b_t x_t = e_1, \tag{2.2}$$

$$d_{t+1} = e_2 + b_{t+1} x_t, \tag{2.3}$$

where $c_t$ is consumption when young, $d_{t+1}$ is consumption when old, $u, v : \mathbb{R}_+ \to [-\infty, \infty)$ are the utility functions for the first and second periods, respectively, $b_t$ is the price of the asset, $x_t$ is asset holdings at the end of period $t$ as well as at the beginning of period $t + 1$, and $\mathbb{E}_t$ is the expectation conditional on the information set in period $t$ (to be specified below).[3] An old agent in period 0 simply consumes all his wealth:

$$d_0 = e_2 + b_0 x_{-1}. \tag{2.4}$$

The market-clearing conditions for the consumption good and the asset are as follows:

$$c_t + d_t = e_1 + e_2, \qquad \forall t \in \mathbb{Z}_+, \tag{2.5}$$

$$x_t = 1, \qquad \forall t \in \mathbb{Z}_+. \tag{2.6}$$

Throughout the paper we assume the following.

**Assumption 2.1.** $u, v : \mathbb{R}_+ \to [-\infty, \infty)$ are continuous, $C^1$ on $(0, \infty)$, and strictly increasing. Furthermore

$$\lim_{c \downarrow 0} u'(c) = \infty. \tag{2.7}$$

[2] See Wigniolle (2014) for a recent extension of Weil's model based on a rank-dependent utility function.

[3] Formally, let $(\Omega, \mathscr{F}, P)$ be a probability space, and let $\{\mathscr{F}_t\}_{t \in \mathbb{Z}_+}$ be a filtration. The conditional expectation $\mathbb{E}_t$ at time $t$ is defined as the expectation conditional on $\mathscr{F}_t$. All stochastic processes indexed by $t \in \mathbb{Z}_+$, including the sunspot processes defined below, are assumed to be adapted to this filtration.

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