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Further results on structural stability and robustness to bounded rationality[☆]

Jian Yu[a,b], Zhe Yang[c,d,*], Neng-Fa Wang[a,e]

[a] Department of Mathematics, Guizhou University, Guiyang 550025, China
[b] Key Laboratory of Games Decision Making and Control Systems, Guizhou Province, Guiyang 550025, China
[c] School of Economics, Shanghai University of Finance and Economics, Shanghai, 200433, China
[d] Key Laboratory of Mathematical Economics (SUFE), Ministry of Education, Shanghai 200433, China
[e] School of Mathematics and Statistics, Guizhou University of Finance and Economics, Guiyang 550025, China

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ABSTRACT

In this paper, we study the model of bounded rationality that has been studied in Anderlini and Canning (2001), Yu and Yu (2006), Yu et al. (2009) and Miyazaki and Azuma (2013). First, using a lower pseudocontinuous rationality function, we prove that the model is structurally stable and robust to ϵ-equilibria for almost all parameter values, and the structural stability implies robustness to bounded rationality. Second, by relaxing the assumption of compactness, if the feasible correspondence is compact-valued and continuous, and the rationality function is continuous, we show that the robustness to ϵ-equilibria implies structural stability. Third, using a lower semicontinuous rationality function, we prove that (λ, ϵ)-stability implies (λ, ϵ)-robustness. Finally, if the feasible correspondence is compact-valued and continuous, and the rationality function is continuous, we obtain that (λ, ϵ)-robustness implies (λ, ϵ)-stability.

1. Introduction

The assumption of full rationality is too strict in many economic models. Ideally, we want a model of bounded rationality in economics. Many studies focused on the experiments rather than theoretical models, see Ido and Roth (1998), Roth (1995). These literatures showed that the assumptions of perfect rationality are inappropriate in analyzing the human behavior. Relaxing the assumption of perfect rationality, a theoretical model of bounded rationality is necessary. In this model, the robustness to bounded rationality is an important issue, that is, we wonder whether the set of equilibria varies continuously as the degree of rationality goes to perfect. Recently, Anderlini and Canning (2001) first established the abstract framework, model M, a parameterized class of general games together with an associated rationality function. To ensure the robustness of equilibria to bounded rationality, Anderlini and Canning (2001) introduced the notion of robustness to ϵ-equilibria if small deviations for the degree of rationality only lead to small variation in the set of equilibria. They also introduced the notion of structural stability, that is, the equilibrium correspondence is a continuous set-valued mapping with respect to the parameter values. They proved an equivalence theorem between the structural stability and robustness to ϵ-equilibria. In this model, the rationality function is a key tool, which is a nonnegative and continuous function whose value is zero under perfect rationality. Many economic models can be studied as the examples of Anderlini and Canning (2001) by defining the corresponding bounded rationality functions. They investigated four examples: finite strategic-form games, pure exchange general equilibrium, a macroeconomic model with strategic complementarities and a rationality expectations macroeconomic model.

Later, Yu and Yu (2006) extended the work of Anderlini and Canning (2001) by relaxing some conditions. Accordingly, they proved that a model M is structurally stable and robust to

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* Corresponding author at: School of Economics, Shanghai University of Finance and Economics, Shanghai 200433, China.
E-mail address: zheyang211@163.com (Z. Yang).

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ϵ-equilibria for almost all parameter values. Yu and Yu (2007) established the connections between bounded rationality and multiobjective games, and obtained some new results for the robustness to ϵ-equilibria and structural stability of multiobjective games and generalized multiobjective games. Furthermore, Yu et al. (2009) obtained the results on the structural stability and robustness to bounded rationality for non-compact cases. Recently, Miyazaki and Azuma (2013) provided a generalization of Anderlini and Canning (2001), Yu and Yu (2006) and Yu et al. (2009), that is, (λ, ϵ)-stable model. They pointed out that (λ, ϵ)-stability implies (λ, ϵ)-robustness, and (λ, ϵ)-stability is equivalent to the essentiality of (λ, ϵ). On the other hand, Miyazaki (2014) introduced the topological robustness to bounded rationality in semialgebraic models. Loi and Matta (2015) increased the complexity in structural stability and robustness to bounded rationality. Applying this abstract construction, they investigated a pure exchange economy.

It is well known that the payoff functions were assumed to be continuous in the Nash equilibrium existence theorems. To study the games with discontinuous payoffs, Morgan and Scalzo (2007) introduced a new class of functions (pseudocontinuous functions) and obtained the maximum theorem with pseudocontinuous functions and the existence theorem of Nash equilibria for games with pseudocontinuous functions. The results on structural stability and robustness to bounded rationality based on the assumptions of lower semicontinuous or continuous rationality function in Anderlini and Canning (2001), Yu and Yu (2006), Yu et al. (2009) and Miyazaki and Azuma (2013). However, the continuity condition of the rationality function is too strict to study some problems that have discontinuous rationality functions. So, we investigate a weaker version of original abstract model using the concept of pseudocontinuous functions, introduced by Morgan and Scalzo (2007).

In this paper, we show the following three results obtained by Anderlini and Canning (2001), Yu et al. (2009) and Miyazaki and Azuma (2013) under weaker conditions. (i) The structural stability implies robustness to bounded rationality under the assumption that the rationality function is lower pseudocontinuous, which is an extension of Theorem 2.2 in Yu et al. (2009). (ii) The robustness to bounded rationality implies structural stability under the noncompactness of the action space and the parameter space. This result is a generalization of Theorem 3.1 in Anderlini and Canning (2001). (iii) (λ, ϵ)-stability implies (λ, ϵ)-robustness if the rationality function is lower semicontinuous. It generalizes Theorem 4.6 in Miyazaki and Azuma (2013). The converse is also proved under the assumptions in (ii). It is a new result for (λ, ϵ)-stable model.

The rest of this paper is organized as follows. In Section 2, we give a brief description of the models of bounded rationality, the definitions and assumptions, and recall some known results. In Section 3, we proceed to give the main results on the relationships between structural stability and robustness, (λ, ϵ)-stability and (λ, ϵ)-robustness.

2. Preliminaries

The model $M = (\Lambda, X, F, R)$ is defined as follows:

- Λ is a parameter space;
- X is a space of actions;
- $F : \Lambda \times X \rightrightarrows X$ is a feasibility correspondence, and F induces a further correspondence $f : \Lambda \rightrightarrows X$ such that

  $f(\lambda) = \{x \in X : x \in F(\lambda, x)\}, \forall \lambda \in \Lambda;$
- the graph of f,

  $Graph(f) = \{(\lambda, x) \in \Lambda \times X : x \in f(\lambda)\}$

  and $R : Graph(f) \longrightarrow \mathbb{R}_+ := \{x \in \mathbb{R} \mid x \geq 0\}$ is a rationality function, which measures the degree of rationality. $R(\lambda, x) = 0$ corresponds to the full rationality;
- for any $\lambda \in \Lambda$ and any $\epsilon \geq 0$, the set of ϵ-equilibria at λ is defined as

  $E(\lambda, \epsilon) = \{x \in f(\lambda) : R(\lambda, x) \leq \epsilon\}$

  and the set of equilibria at λ is defined as

  $E(\lambda) = E(\lambda, 0) = \{x \in f(\lambda) : R(\lambda, x) = 0\}.$

Definition 2.1. The model M is robust to ϵ-equilibria at $\lambda \in \Lambda$ if for any $\delta > 0$, there exists $\bar{\epsilon} > 0$ such that

$h(E(\lambda', \epsilon), E(\lambda')) < \delta$

for any $(\lambda', \epsilon) \in \Lambda \times \mathbb{R}_+$ with $\epsilon < \bar{\epsilon}$ and $\rho(\lambda, \lambda') < \bar{\epsilon}$, where h is the Hausdorff distance defined on X.

Definition 2.2. The model M is structurally stable at $\lambda \in \Lambda$ if the equilibrium correspondence $E : \Lambda \rightrightarrows X$ is continuous at $\lambda \in \Lambda$.

To study the robustness of (λ, ϵ)-stable model, Miyazaki and Azuma (2013) extended the notions of structural stability and robustness to bounded rationality as follows.

Definition 2.3. The model M is (λ, ϵ)-robust if for any $\delta > 0$, there exists $\bar{\epsilon} > 0$ such that

$h(E(\lambda', \epsilon), E(\lambda', \epsilon')) < \delta$

for any $(\lambda', \epsilon') \in \Lambda \times \mathbb{R}_+$ with $|\epsilon - \epsilon'| < \bar{\epsilon}$ and $\rho(\lambda, \lambda') < \bar{\epsilon}$, where h is the Hausdorff distance defined on X.

Definition 2.4. The model M is (λ, ϵ)-stable if the equilibrium correspondence $E : \Lambda \times \mathbb{R}_+ \rightrightarrows X$ is continuous at $(\lambda, \epsilon) \in \Lambda \times \mathbb{R}_+$.

We next recall some known results concerning correspondences from Aliprantis and Border (2006). Let X, Y be two metric spaces. A correspondence $T : Y \rightrightarrows X$ is said to be (i) upper semicontinuous at y if for any open O in X with $T(y) \subset O$, there exists an open neighborhood U of y such that $T(y') \subset O$ for any $y' \in U$; (ii) upper semicontinuous on Y if T is upper semicontinuous at each $y \in Y$; (iii) lower semicontinuous at $y \in Y$ if, for any open subset O of X with $O \cap T(y) \neq \emptyset$, there exists an open neighborhood $U(y)$ of y such that $O \cap T(y') \neq \emptyset$ for any $y' \in U(y)$; (iv) lower semicontinuous on Y if T is lower semicontinuous at each $y \in Y$; (v) closed if $Graph(T) = \{(y, x) \in Y \times X | x \in T(y)\}$ is closed in $Y \times X$.

The following Lemmas 2.1 and 2.2 (see Theorem A.1.1 in Florenzano, 2003) will play a crucial role in this paper.

Lemma 2.1. *Let $f : Y \rightrightarrows X$ be a correspondence, if $f(y)$ is compact, then f is upper semicontinuous at y if and only if for all sequences $\{y_n\}, \{x_n\}$ such that $y_n \to y, x_n \in f(y_n)$, there exists a subsequence $\{x_{n_k}\}$ of $\{x_n\}$ such that $x_{n_k} \to x \in f(y)$.*

Lemma 2.2. *A correspondence $f : Y \rightrightarrows X$ is lower semicontinuous at y if and only if for all sequences $\{y_n\}$ convergent to y, and all $x \in f(y)$, there exists a sequence $\{x_n\} \subset X$ such that $x_n \in f(y_n)$ for each n and $x_n \to x$.*

The following Lemma 2.3 is due to Theorem 2 in Fort (1951), also see Lemma 2.1 in Yu (1999).

Lemma 2.3. *Let Y be a complete metric space, X be a metric space, and $F : Y \rightrightarrows X$ be a nonempty, compact-valued and upper semicontinuous correspondence. Then there exists a dense residual subset Q of Y such that F is continuous at every $y \in Q$.*