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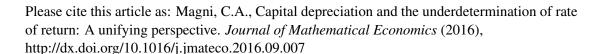Please cite this article as: Magni, C.A., Capital depreciation and the underdetermination of rate of return: A unifying perspective. *Journal of Mathematical Economics* (2016), http://dx.doi.org/10.1016/j.jmateco.2016.09.007

Capital depreciation and the underdetermination of rate of return: A unifying perspective

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Abstract. This paper shows that the notion of rate of return is best understood through the lens of the average-internal-rate-of-return (AIRR) model, first introduced in Magni (2010a). It is an NPV-consistent approach based on a *coherent* definition of rate of return and on the notion of *Chisini mean*, it is capable of solving the conundrums originated by the rate-of-return notion and represents a unifying theoretical paradigm under which every existing measure of wealth creation can be subsumed. We show that a rate of return is *underdetermined* by the project's cash-flow stream; in particular, a unique return function (not a unique rate of return) exists for every project which maps depreciation classes into rates of return. The various shapes a rate of return can take on (internal rate of return, average accounting rate of return, modified internal rate of return etc.) derive from the (implicit or explicit) selection of different depreciation patterns. To single out the appropriate rate of return for a project, auxiliary assumptions are needed regarding the project's capital depreciation. This involves value judgment. On one side, this finding opens terrain for a capital valuation theory yet to be developed; on the other side, it triggers the creation of a toolkit of domain-specific and purpose-specific metrics that can be used, jointly or in isolation, for analyzing the economic profitability of a given project. We also show that the AIRR perspective has a high explanatory power that enables connecting seemingly unrelated notions and linking various disciplines such as economics, finance, and accounting. Some guidelines for practitioners are also provided.

Keywords. Rate of return, depreciation, capital, underdetermination, NPV, AIRR, mean, coherent.

1 Introduction

This paper shows that the notion of rate of return is *unique* but the rate of return of an economic transaction is *not unique*. In particular, the paper uses a coherence principle and the notion of *Chisini mean* to establish an underdetermination principle, according to which a rate of return cannot be singled out from a cash-flow stream as such. Rather, any cash-flow stream is associated with a *unique* return function which maps capital values into rates of return, so that infinitely many combinations of capital and rate generate the same net present value (NPV). A rate of return is singled out only if a well-defined statement upon capital depreciation is made by the analyst, implicitly or explicitly. The resulting approach is the so-called average internal rate of return (AIRR), which has been introduced in Magni (2010a) and developed in a vast array of subsequent papers (e.g., Magni 2011a, 2013a,b 2014a,b, 2015a).

The AIRR is a theoretical paradigm based on a mathematically simple and economically meaningful model, which coherently defines a rate of return as a growth rate of capital (and, therefore, as a *ratio*). The notion of Chisini mean enables fulfilling this 'coherence' for multiperiod projects as well as one-period projects, so a rate of return for a multiperiod project can be viewed as a generalized weighted mean of constituent one-period rates. This unique structure makes possible a deeper analysis of economic profitability that the traditional NPV analysis cannot accomplish and is capable of subsuming every single profitability measure (NPV included) into a unifying framework.