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Feasible sets, comparative risk aversion, and comparative uncertainty aversion in bargaining

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ABSTRACT

We study feasible sets of the bargaining problem under two different assumptions: the players are subjective expected utility maximizers or the players are Choquet expected utility maximizers. For the latter case, we consider the effects on bargaining solutions when players become more risk averse and when they become more uncertainty averse.

1. Introduction

In the classical bargaining problem of Nash (1950) a number of bargainers face the task of finding a unanimous agreement over the expected utility allocations resulting from the lotteries, i.e. finite probability distributions, over a set of alternatives. In this paper we extend the bargaining problem by adopting the Savage (1954) framework of decision making under uncertainty. We study the consequences for the feasible set of a bargaining problem under two different assumptions about the way bargainers make decisions: as subjective expected utility maximizers or as Choquet expected utility maximizers. In the latter case, the preferences of a bargainer are characterized by a utility function for riskless alternatives and lotteries, and a capacity (non-additive probability measure) for uncertain states of the world. Thus, the attitude of a bargainer towards risk can be strictly separated from his attitude towards uncertainty, and we use this feature to derive results about the effects of comparative risk aversion and comparative uncertainty aversion on the outcomes assigned by specific (namely, monotonic) bargaining solutions.

The applications that we have in mind concern bargaining situations where the agreement that the parties reach may be contingent on the future state of the world that will materialize. For instance, the wages and labor conditions in an agreement in the traditional bargaining problem between an employer and a labor union may be made dependent on the future state of the economy.

1.1. Related literature

Most closely related to the present work is Köbberling and Peters (2003). In that paper, the bargaining problem is extended by assuming that probabilities in lotteries may be transformed by a probability weighting function, and overall utilities are rank dependent (Quiggin, 1982). The paper distinguishes between (comparative) utility risk aversion as expressed by concavity of the utility function for lotteries and (comparative) probabilistic risk aversion as expressed by convexity of the probability weighting function. The effects of these attitudes on bargaining solutions are in line with what we find in the present paper.

There are several papers, notably Safra and Zilcha (1993), Volij and Winter (2002), and Rubinstein et al. (1992), which study the qualitative predictions of the Nash bargaining solution beyond the expected utility framework. Roth and Rothblum (1982)[1] show that under the Nash bargaining solution, it is disadvantageous to play against a more risk averse opponent, even in the case of risky outcomes, provided that lotteries only attach positive probabilities to

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[1] For other results on comparative risk aversion in the classical bargaining model under expected utility see Kannai (1977), Kihlstrom et al. (1981), Wakker et al. (1986), and Safra et al. (1990).

alternatives that are preferred over the riskless disagreement alternative. Safra and Zilcha (1993) show that this result breaks down if the expected utility assumption is abandoned. This is confirmed by results in the present paper (see Example 5.7). Volij and Winter (2002) show that increased risk aversion of one bargainer is beneficial for that bargainer and hurts his opponent, given that both bargainers are risk loving (so that the outcome is a lottery) in a model adopting Yaari's (1987) dual theory of choice under risk. On the other hand, Rubinstein et al. (1992) find that under their extension of the Nash bargaining solution it is disadvantageous for a bargainer to be more risk averse, a result which is not confirmed in our framework (see again Example 5.7). A prudent conclusion that may be drawn from all these results is that the Nash bargaining solution behaves irregularly under changes in risk attitudes, and that in this respect it is also sensitive to the way in which risk and uncertainty are modeled beyond the expected utility assumption. As will appear again in the present paper (but for instance also in Safra and Zilcha, 1993, or Köbberling and Peters, 2003) bargaining solutions that are monotonic tend to behave much more regularly under changes in attitude towards risk and uncertainty. Examples of such solutions are the Kalai–Smorodinsky (Raiffa, 1953; Kalai and Smorodinsky, 1975) and egalitarian (Kalai, 1977) solutions.

1.2. This paper

The set of riskless alternatives in this paper is the set of all divisions of one unit of a perfectly divisible good among n bargainers or players. Each player has a strictly increasing and concave utility function which depends only on the own share of the good. Lotteries over riskless alternatives are included as well, and under certainty the utility of a lottery is its expected utility. Further, a finite set of states of the world is assumed.

In Section 3 we follow Savage (1954) and assume that each player attaches subjective utilities to the states of the world. An act assigns to each state of the world a riskless alternative or lottery. We show that the feasible set resulting from considering all possible acts is comprehensive, compact and convex, and strictly comprehensive if each player attaches positive probability to each state of the world.

The results in Section 3 form the basis for those in Section 4, where we assume that the players' probability assessments over the states of the world can be non-additive, and are described by capacities; players are then assumed to be Choquet expected utility maximizers (Schmeidler, 1986, 1989). We show that in this case the feasible set is convex if the capacities are convex. Moreover, we define and characterize increased uncertainty aversion in terms of capacities and increased risk aversion in terms of the utility functions for lotteries.

In Section 5, we show that for a monotonic bargaining solution, increased uncertainty aversion of a player is disadvantageous for the opponents, whereas increased risk aversion is advantageous for the opponents. We also show that a more uncertainty averse player prefers his allocation to the one obtained by his less uncertainty averse alter ego; whereas a less risk averse player prefers his allocation to the one obtained by his more risk averse alter ego. For non-monotonic bargaining solutions these results may break down, as we show by examples for the Nash bargaining solution.

Section 6 concludes.

2. Preliminaries

Following Savage (1954), the analytical framework of decision making under uncertainty involves a set of *states of the world*, a set of *consequences*, and a set of *acts*—i.e., functions that map states of the world to consequences. Resolution of uncertainty reveals the unique true state of the world, and thus, given an act, provides a decision maker with certainty about the realized consequence.

The situation under consideration in this paper is one with multiple decision makers, or *players*. Specifically, the player set is $N = \{1, \ldots, n\}$ where $n \in \mathbb{N}$, $n > 1$. The set of states of the world is $\Omega = \{1, \ldots, K\}$ where $K \in \mathbb{N}$, $K > 1$. The set of *consequences* is the set of (simple) lotteries – probability distributions with finite support – on the set

$$A = \left\{ a \in \mathbb{R}^n_+ \mid \sum_{i=1}^n a_i \leq 1 \right\}$$

of all possible divisions of one unit of a perfectly divisible good among the n players[2]; more formally, the set of consequences is:

$$L = \left\{ \ell : A \to [0,1] \mid \{a \mid \ell(a) > 0\} \text{ is finite}, \sum_{a \in A} \ell(a) = 1 \right\}.$$

Every lottery $\ell \in L$ induces a probability distribution with finite support over player i's consumption space and, hence, induces a simple lottery $l_i(\cdot; \ell) : [0,1] \to [0,1]$ defined by

$$l_i(x_i; \ell) = \sum_{a \in A:\ a_i = x_i} \ell(a)$$

for every share $x_i \in [0,1]$ of the good. An *act* is a map f assigning to every $k \in \Omega$ an element of A or a lottery $\ell \in L$. The set of all acts is denoted by F.

Player i's preferences over acts F are represented by a (reflexive) complete and transitive binary relation $\succcurlyeq_i \subseteq F \times F$. As usual, we write $f \succcurlyeq_i g$ instead of $(f, g) \in \succcurlyeq_i$. For each player i, a function U_i, mapping acts into the reals, *represents* i's preferences $\succcurlyeq_i$ over F if for all $f, g \in F$, we have $U_i(f) \geq U_i(g)$ if and only if $f \succcurlyeq_i g$. In this paper, we consider different specifications of the functions U_i, and for these specifications examine the properties of the set

$$S = \{(U_1(f), \ldots, U_n(f)) \mid f \in F\}.$$

For the sake of brevity, we write $U(f) \equiv (U_1(f), \ldots, U_n(f))$ for every $f \in F$.

3. Subjective expected utility

In this section, the players in N are assumed to be *Subjective Expected Utility* (SEU) maximizers (Savage, 1954). The premise of the SEU model is that each player i evaluates an act $f \in F$ based on his subjective belief about the states of the world. Specifically, player i's evaluation of an act $f \in F$ is given by

$$U_i(f) = \sum_{k \in \Omega} \pi_i(k)\, u_i(f_i(k)),$$

where $\pi_i : 2^{\Omega} \to [0,1]$ represents his subjective probability distribution over the state space Ω,[3] and the continuous, strictly increasing, and concave function $u_i : [0,1] \to \mathbb{R}$ is player i's expected utility function for lotteries: if $f(k)$ is a lottery then with some abuse of notation $u_i(f_i(k))$ denotes player i's expected utility for the lottery $f_i(k) = l_i(\cdot; f(k))$ resulting from $f(k)$. Since the representation of a player's preference is unique up to positive linear transformations, we may assume without loss of generality that $u_i(0) = 0$ and $u_i(1) = 1$ for every $i \in N$.

[2] Here, $\mathbb{R}^n_+ = \{x \in \mathbb{R}^n \mid x \geq 0\}$. We use the following vector inequalities: for all $x, y \in \mathbb{R}^n$, $x \geq y$ if $x_i \geq y_i$ for each i, $x > y$ if $x \geq y$ and $x \neq y$, and $x \gg y$ if $x_i > y_i$ for each i.

[3] More precisely, π_i is a probability measure: (i) $\pi_i(\emptyset) = 0$ and $\pi_i(\Omega) = 1$; (ii) for all $E \subseteq \Omega$, $\pi_i(E) = \sum_{k \in E} \pi_i(k)$.

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