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Evaluation of long-dated assets: The role of parameter uncertainty ☆

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ABSTRACT

Under expected utility, the uncertainty that affects the parameters of the random walk of consumption growth has no effect on the value of short-term claims and makes the term structure of risk-free rates decreasing. The term structure of aggregate risk premia is increasing when the uncertain cumulants of log consumption are independent. We apply these generic results to the case of an uncertain probability of catastrophes, and to the case of an uncertain trend or volatility of growth. Adding some persistence to unobservable shocks into our benchmark model, we show that the term structure of risk premia is hump-shaped.

1. Introduction

Do we do enough for the distant future? This question is implicit in many policy debates, from the fight against climate change to the speed of reduction of public deficits, investments in research and education, or the protection of the environment and of natural resources for example. The dual question is whether markets value assets with extra long-term cash-flows in the right way, thereby providing the efficient price signals to economic agents to invest for the long run. The discount rate used to evaluate investments is the key determinant of our individual and collective efforts in favor of the future. In relation to this question, an intense debate has emerged among environmental and climate economists since Weitzman (1998) about whether one should use different discount rates for different time horizons t.

A critical dimension to this problem is the deep uncertainties that surround the dynamics of economic growth. For example, a relatively small change in the permanent trend of growth has an immense impact on future consumption when projected over many decades or centuries. Similarly, the uncertainty about the true volatility of growth magnifies long-run risks, as does the ambiguity surrounding the probability of rare macroeconomic catastrophes. Our aim in this paper is to provide a systematic analysis of the impact of parametric uncertainty on asset prices in the standard consumption-based

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CAPM. It is shown that the parameter uncertainty that affects the random walk of the consumption growth rate has no effect on the price of short-term bond and consumption claims. [1] For longer maturities, our benchmark is a model in which the term structures are known to be flat in the absence of parameter uncertainty. Under the discounted expected utility model with constant relative risk aversion, this requires the growth of log consumption to be governed by a random walk – meaning that increments are stationary and serially independent. Under this assumption, the growth process is characterized by the distribution of increments in log consumption. This distribution is subject to some parameter uncertainty.

Our main results are generic and hold without making any restriction on this distribution or on the nature of the parameter uncertainty. It can include ambiguity related to the trend of growth, the volatility of growth, or the frequency of macro catastrophes for example. The mathematical methodology used in this paper is based on the Cumulant-Generating Function (CGF) associated to the distribution of log consumption. Martin (2013) recently used the properties of CGF to characterize asset prices when the growth of log consumption is not Gaussian. This paper provides various illustrations of the power of this method which we improve in the context of parameter uncertainty. Under our assumptions, the risk-free rate and the risk premium can be obtained by performing sequences of two CGF calculations, the first on the conditional increment of log consumption, and the second on the conjugate distribution of the uncertain parameters.

As is well-known (see e.g. Billingsley, 1995), a probability distribution can be represented by its vector of cumulants, so that the parameter uncertainty affecting the distribution of increments can be characterized by the joint distribution of its cumulants. It is shown that parameter uncertainty has *no* effect on short-term risk-adjusted discount rates. To be more precise, the efficient instantaneous risk-adjusted discount rates are obtained by applying the standard CAPM pricing formulas using the expected cumulants as if they would be the true values. In words, this implies that mean-preserving spreads in the distribution of cumulants have no impact on the price of very short-term zero-coupon bonds and equity in our framework. This generalizes a result by Veronesi (2000) who has demonstrated that the short-term risk-free interest rate is not affected by parameter uncertainty when this uncertainty affects the drift rate of aggregate consumption, i.e. when only the first cumulants is uncertain.[2] This is due to the fact that the uncertainty affecting the expected growth has no impact on the conditional posterior volatility of consumption in the short run. This result is also reminiscent of a result by Hansen and Sargent (2010) who showed that parameter uncertainty does not contribute to local uncertainty prices in a Bayesian analysis.

Gollier (2008, 2012) shows that the term structure of interest rates is decreasing in the standard consumption-based CAPM with a prudent representative agent whenever the growth rates of consumption are positively serially correlated, i.e., when shocks to the growth rate are persistent. This is because the persistence of shocks tends to magnify the uncertainty affecting long-run consumption, thereby inducing the prudent representative agent to favor investments yielding sure benefits for the distant future. This reduces the long-term interest rate. Our generic model with parametric uncertainty is a special case since, under parametric uncertainty, the observation of a large growth rate in the short run yields a permanent optimistic revision of beliefs. As observed by Collin-Dufresne al. (2016), posterior moments of growth are martingales under rational expectations, so that shocks to beliefs about the distribution of increments in log consumption are permanent. This magnifies the long-run consumption risk.

Adjusting the discount rates to risk requires estimating the maturity-specific risk premia. Because the persistence of learning shocks magnifies the long-run aggregate risk compared to this benchmark case, it raises the aggregate long-term risk premium. This provides an intuition to our result that the term structure of the aggregate risk premia is increasing. This property holds if the set of uncertain cumulants are independently distributed. However, this property does not hold in general. This is because the risk premium is also affected by the higher cumulants of log consumption outside the Gaussian world. For example, if the trend and the volatility of growth are negatively correlated, then the term structure of the annualized skewness of log consumption is locally decreasing. This decreasing skewness effect can be stronger than the increasing variance effect to make the term structure of risk premia decreasing.

The persistence of shocks to beliefs has an ambiguous effect on the long-term risk-adjusted discount rate because it reduces the risk-free rate and it potentially raises the risk premium. If the asset's beta is large enough, the net effect may be positive, yielding an increasing term structure of the risk-adjusted discount rates. If the parameter uncertainty affects cumulants in a statistically independent way, we show that the risk-adjusted discount rate is reduced by the uncertainty if and only if the consumption CAPM beta of the asset is smaller than half the degree of relative risk aversion. This suggests that parameter uncertainty should induce us to invest more for the distant future if the investment opportunity set contains enough projects with a small beta.

These generic results are presented in Section 3. Following the current trend of the literature, we explore in Section 4 and 5 some special cases to quantify these effects. In Section 4, it is assumed that the economy may face macroeconomic catastrophes at low frequency. In normal time, the growth of log consumption is Gaussian, but a large drop in aggregate consumption strikes the economy at infrequent dates. Our modeling duplicates the one proposed by Barro (2006, 2009), except for the recognition of the existence of some parametric uncertainty. Martin (2013) convincingly demonstrates that it is complex to estimate the true probability of infrequent catastrophes, and that a small modification in the choice of

[1] Brennan (1997) was the first to examine the term structure of risk premia. Ang and Liu (2004) refer to the notion of "spot discount rates", which is parallel to the "prices of zero-coupon equity" and "dividend strips" in Lettau and Wachter (2007).

[2] Veronesi (2000) considers a Markov multiple-regime switch process for the growth trend. We refer here to the special case in which the growth trend is permanent.