Computers and Structures 131 (2014) 1–11

Contents lists available at ScienceDirect

Computers and Structures

journal homepage: www.elsevier.com/locate/compstruc

A bi-level hierarchical method for shape and member sizing optimization of steel truss structures

Forest Flager [a,*], Akshay Adya [b], John Haymaker [c], Martin Fischer [d]

[a] Stanford University, CIFE, 473 Via Ortega, Room 275B, Stanford, CA 94305-4020, United States
[b] Bentley Systems, 1600 Riviera Avenue Suite 300, Walnut Creek, CA 94596, United States
[c] Schools of Architecture and Building Construction, Georgia Tech, 280 Ferst Dr., First Floor, Atlanta, GA 30332, United States
[d] Stanford University, CIFE, 473 Via Ortega, Room 297, Stanford, CA 94305-4020, United States

ARTICLE INFO

Article history:
Received 13 October 2011
Accepted 7 October 2013
Available online 13 November 2013

Keywords:
Structural optimization
Size optimization
Shape optimization
Steel structures
Member sizing
Truss

ABSTRACT

This paper describes a new bi-level hierarchical method for optimizing the shape and member sizes of both determinate and indeterminate truss structures. The method utilizes a unique combination of algorithms that are organized hierarchically: the Fully Constrained Design (FCD) method for discrete sizing optimization is nested within SEQOPT, a gradient-based optimization method that operates on continuous shape variables. We benchmarked the method against several existing techniques using numerical examples and found that it compared favorably in terms of solution quality and computational efficiency. We also present a successful industry application of the method to demonstrate its practical benefits.

1. Introduction

Engineers often are challenged to design steel truss structures that are both economical and reliable. The design process involves specifying each of the following three aspects of the structure: (i) *topology*, which concerns the number and connectivity of members; (ii) *shape*, which pertains to the location of structural joints;and (iii) *sizing*, which involves defining member cross-sections [1]. The specification of each aspect of the structure typically corresponds to the three major stages of the engineering design process as defined by Pahl and Beitz [2]: conceptual, embodiment (design development) and detail. The topology of the structure is typically identified during conceptual design based on the functional requirements and architectural aesthetics, whereas the structure's shape and member sizing are determined during the design development and detailed design phases, respectively.

This paper presents a bi-level hierarchical method with a unique combination of algorithms to optimize the latter two aspects of the structure – shape and sizing – given a fixed topology. The objective of the optimization is to minimize the cost of the structure, while satisfying design performance requirements for safety and serviceability. In this case, the cost of the structure is estimated by multiplying the total steel weight by the price per unit. Steel weight is commonly used as a proxy for cost, provided that industry standard means and methods of construction are employed [3].

We treat shape variables as continuous in this investigation, meaning that any value can be assumed within the specified limits (e.g., allowing the depth of a truss to assume any value between, for example, 900 and 1800 mm). Sizing variables, on the other hand, are discrete, meaning that only certain specified values can be assumed. This is consistent with industry practice where engineers commonly select structural member sizes from a set of standard steel profiles that are mass-produced in specific sizes (e.g., W14 × 132, W14 × 120, etc.) [4]. Typically, there is a cost premium and/or quantity requirement associated with using steel profiles that do not conform to these standard sizes [5].

Traditionally, shape and sizing optimization has been an iterative process that is performed manually by the engineer. The first step in the process is usually to define the initial shape and sizing configuration of the structure based on architectural requirements, engineering rules of thumb, and past experience. Next, an analytical model is created that includes an idealized representation of the structure's topology, shape, member sizes, and loading. The analytical model is used to calculate the structure's response to the defined loading (e.g., forces, deflections). These responses are then checked against the design requirements for safety and serviceability. Finally, the engineer reviews the results and may

* Corresponding author. Tel.: +1 415 728 7197.
E-mail addresses: forest@stanford.edu, fflager@gmail.com (F. Flager), akshay.adya@alumni.stanford.edu (A. Adya), johnrhaymaker@gmail.com (J. Haymaker), fischer@stanford.edu (M. Fischer).

0045-7949/$ - see front matter
http://dx.doi.org/10.1016/j.compstruc.2013.10.004

elect to modify either the shape of the structure or the sizes of constituent members.

The number of possible shape and sizing configurations for a given design problem is termed the *design space* [6]. The size of the design space is an exponential function of the number of design variables and the number of possible choices for each variable. For example, a problem with x variables and n discrete choices per variable has $n[x]$ possible configurations. The size of the design space for most problems encountered in industry is so large that it is impractical to explore all possible design alternatives [7]. Engineers using the manual methods described above customarily have time to evaluate only a few design alternatives [8]. Vast areas of the design space are, therefore, left unexplored even though they may contain better performing shape and sizing configurations [9].

Numerous formal optimization methods have been developed to improve upon traditional approaches by reducing design iteration time, thereby enabling the evaluation of a greater number of design alternatives that can lead to better quality solutions. The majority of formal methods surveyed consider only member sizing design variables [10]. The inherent coupling between size and shape variables, however, makes it more advantageous to consider both variable types simultaneously [11]. In Section 2, we survey existing shape and sizing optimization methods and discuss their respective strengths and limitations with regard to generality and efficiency.

The goal of the research presented in this paper was to develop a formal optimization method that (i) can generally be applied to problems with a mix of discrete sizing and continuous shape variables and (ii) efficiently handles large variable sets that are typically encountered in industry. To achieve these objectives, the proposed method employs different optimization algorithms to operate on discrete sizing and continuous shape variables as discussed in Section 3. In Section 4, we benchmark this unique combination of algorithms against other leading approaches using two standard numerical examples. In Section 5, we present a successful industry implementation of the method on two large stadium roof trusses. Finally, in Section 6 we summarize the lessons learned and discuss the method's suitability for general industry application.

2. Shape and sizing optimization

Methods for shape and sizing optimization of trusses and frames generally can be categorized as either single-level or multilevel depending upon how the problem is decomposed.

2.1. Single-level methods

Most structural optimization methods described in the literature are single-level approaches because a single optimization algorithm is used to operate on shape and sizing variables simultaneously. Although the analysis may be distributed, all design decisions are made by a single optimization algorithm. Both deterministic and heuristic single-level methods are described and their respective limitations discussed below.

The deterministic methods that have received the most attention in the research community are stress-ratio (or fully stressed design), linear programming, nonlinear programming, and branch and bound methods [12]. The stress-ratio method seeks to proportion each member of a structure so that it is loaded to the maximum safe performance limit under the action of at least one of the applied load cases. This approach is applicable to stress and local buckling constrained structures. While the solution quality of the stress-ratio method has been shown to be sub-optimal and highly dependent on the start point of the optimization process [13,14], the method has been widely adopted in professional practice due to its simplicity in concept and implementation. The stress-ratio method may be considered to be part of the optimality criterion approach to structural design, and this more general concept has been the subject of considerable research for many years [15–17].

Linear programming was first applied to unconstrained shape and sizing optimization problems involving plane trusses subject to a single loading case [18]. A penalty function method was later developed and successfully applied to various constrained truss problems considering a cost objective function [19]. Sequential Linear Programming (SLP) methods have been applied to problems with multiple load cases and constraints on eigenfrequencies [20]. Linear programming approaches, however, result in severe approximation errors when applied to problems with nonlinear responses [21]. To reduce these errors, researchers developed an augmented Lagrange multiplier method that utilizes second order Taylor series expansions to express stress and displacement quantities in terms of shape and sizing variables [22,23]. The efficiency of the method was later improved by using Taylor series expansions to approximate forces, rather than stresses and displacements [11].

The deterministic methods described above require the first derivative of the objective and constraint functions with respect to the design variables. Therefore, these methods are not readily applicable to problems where the objective and/or constraint functions are discontinuous or are not easily expressed in terms of the design variables [24]. These methods also assume continuity of the design variables. When a discrete solution is required, approximation techniques are used to generate discrete variable values from the continuous results. Researchers have shown that these approximations can result in solutions that are sub-optimal or even infeasible [21].

The classical branch and bound method was originally developed for linear problems [25], but has been subsequently adapted to nonlinear problems [26]. Compared to the techniques discussed above, this method is known to generate superior quality solutions at the expense of computational efficiency [27]. Various approaches for approximating structural responses have been tested to improve the computational efficiency of the method, but branch and bound remains more expensive than comparable deterministic approaches [27,28].

In recent years, there has been significant research on the application of heuristic techniques to structural shape and sizing problems, including genetic algorithms [24,29–31], simulated annealing [32], and evolutionary strategies [33]. These methods are capable of handling both discrete and continuous variables simultaneously, and there is no limitation on the continuity of the search space. Researchers have also demonstrated that heuristic techniques such as genetic and evolutionary algorithms can be applied to conceptual structural design problems involving topology as well as shape and sizing variables [34,35]. These combined topology, shape and sizing methods allow for human input to guide the optimization by manipulating the algorithm parameters during the iterative design generation and analysis process. A disadvantage of these heuristic methods is that they compare unfavorably to the deterministic methods discussed above in terms of computational efficiency [36].

2.2. Multilevel methods

Multilevel formulations employ more than one optimization algorithm, with each algorithm operating on a specific set of variables. Relatively few multilevel methods have been applied to optimize the shape and sizing of truss and frame structures. Vanderplaats and Moses developed the alternating gradient method [37] that decomposes the problem into two separate, but