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A new beam element with transversal and warping eigenmodes

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ABSTRACT

In this work, we present a new formulation of a 3D beam element, with a new method to describe the transversal deformation of the beam cross section and its warping. With this new method we use an enriched kinematics, allowing us to overcome the classical assumptions in beam theory, which states that the plane section remains plane after deformation and the cross section is infinitely rigid in its own plane. The transversal deformation modes are determined by decomposing the cross section into 1D elements for thin walled profiles and triangular elements for arbitrary sections, and assembling its rigidity matrix from which we extracts the Eigen-pairs. For each transversal deformation mode, we determine the corresponding warping modes by using an iterative equilibrium scheme. The additional degree of freedom in the enriched kinematics will give rise to new equilibrium equations, these have the same form as for a gyroscopic system in an unstable state, these equations will be solved exactly, leading to the formulation of a mesh free element. The results obtained from this new beam finite element are compared with the ones obtained with a shell model of the beam.

1. Introduction

The classical beam theories are all based on some hypothesis that are sufficient in most cases for structure analysis, but fail in more complex cases to give accurate results and can lead to non-negligible errors. For Timoshenko beam theory, widely used by structural engineers, two assumptions are made, the cross section remains plane after deformation and every section is infinitely rigid in its own plane, this means that the effects of warping shear lag and transversal deformation are neglected, these phenomenon are important in bridge study, especially when dealing with bridge with small width/span ratio, and with thin walled cross section.

The problem of introducing the warping effect into beam theory has been widely treated. The most classical approach is to introduce extra generalized coordinates, associated with the warping functions calculated from the Saint–Venant solution, which is exact for the uniform warping of a beam, but gives poor results in the inverse case, especially near the perturbation where the warping is restrained. Bauchau [1], proposes an approach that consists in improving the Saint–Venant solution, that considers only the warping modes for a uniform warping, by adding new eigenwarping modes, derived from the principle of minimum potential energy. Sapountzakis and Mokos [2,3] calculate a secondary shear stress, due to a non-uniform torsion warping, this can be considered as the derivation of the second torsion warping mode in the work of Ferradi et al. [4], where a more general formulation is given, based on a kinematics with multiple warping eigenmodes, obtained by considering an iterative equilibrium scheme, where at each iteration, equilibrating the residual warping normal stress will lead to the determination of the next mode, this method has given very accurate results, even in the vicinity of a fixed end where the condition of no warping is imposed.

The aim of this paper is to propose a new formulation, which not only takes into account the warping of the cross section, but also its transversal deformation, an element of this type falls in the category of GBT (generalized beam theory), which is essentially used to study elastic buckling of thin walled beam and cold formed steel members [5,6], this is done by enriching the beam's kinematics with transversal deformation modes, and then determining the contribution of every modes to the vibration of the beam. In the formulation developed by Ferradi et al. [4], a series of warping functions are determined, associated to the three rigid body motions of horizontal and vertical displacements and torsion, which can be considered as the three first transversal deformation modes. The main idea of this article is to go beyond these three first modes, and determine a series of new transversal deformation modes, calculated for an arbitrary cross section, by modeling this section with triangular or/and 1D element, assembling its rigidity matrix and extracting the eigenvalues and the corresponding eigenvectors, for a desired number of modes. Then, for each determined mode, we will derive a series of warping functions, noting that we will need at least one to represent exactly the case of uniform warping in the beam. With all these additional transversal and warping modes, we will obtain

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an enhanced kinematics, capable of describing accurately, arbitrary displacement and stress distribution in the beam. Using the principal of virtual work we will derive the new equilibrium equations, which appear to have the same form as the dynamical equations of a gyroscopic system in an unstable state. Unlike classical finite element formulation, where interpolation functions are used for the generalized coordinates, we will perform for this formulation, as in [4], an exact solution for the arising differential equations system, leading to the formulation of a completely mesh free element.

The results obtained from the beam element will be compared to those obtained from a shell (MITC-4) and a brick (SOLID186 in Ansys™) model of the beam. Different examples are presented to illustrate the efficiency and the accuracy of this formulation.

2. Determination of transversal deformation modes

For an arbitrary beam cross section, composed of multiple contours and thin walled profiles, to calculate the transversal deformation mode, we use a mesh with triangular elements for the 2D domain delimited by some contours and beam element for the thin walled profiles. As for a classical structure with beam and shell elements, we assemble the rigidity matrix $\boldsymbol{K_s}$ for the section, by associating to each triangular and beam element a rigidity matrix (see Appendix A.1) calculated for a given thickness. We calculate the eigenvalues and their associated eigenvectors of the assembled rigidity matrix of the section, by solving the standard eigenvalue problem (SEP):

$$\boldsymbol{K}_s \boldsymbol{v} = \lambda \boldsymbol{v} \tag{1a}$$

We note that for all that will follow, if it's written in bold, a lowercase letter means a vector and an uppercase letter means a matrix.

The strain energy associated to a transversal mode represented by its Eigen-pair (λ,$\boldsymbol{v}$) will be given by:

$$U = \frac{1}{2}\boldsymbol{v}^T \boldsymbol{K}_s \boldsymbol{v} = \frac{1}{2}\lambda \boldsymbol{v}^T \boldsymbol{v} = \frac{1}{2}\lambda \tag{1b}$$

Thus, the modes with the lowest eigenvalues mobilize less energy and then have more chances to occur. From the resolution of the SEP, in Eq. (1a) we obtain a set of vectors that we note $\boldsymbol{\psi}^i = \left(\psi_y^i, \psi_z^i\right)$, where ψ_y^i and ψ_z^i are the vertical and horizontal displacement, respectively, for the ith transversal deformation mode (Figs. 1 and 2). We note that the three first modes with a zero eigenvalue, corresponds to the classical modes of a rigid body motion:

$$\boldsymbol{\psi}^1 = (1,0), \quad \boldsymbol{\psi}^2 = (0,1), \quad \boldsymbol{\psi}^3 = (-(z-z_0), y-y_0) \tag{2}$$

where (y_0,z_0) are the coordinates of the torsion center of the section.

In our formulation, the only conditions that needs to be satisfied by the set of transversal modes functions, is that they are linearly independent, not necessarily orthogonal. Thus, an important feature of our formulation is that any linearly independent set of functions can be used to enrich our kinematics, the resolution of the differential equation system, performed later, being completely independent from the choice of the transversal and warping modes functions. In our case, the condition of linear independency is satisfied by construction, from the solution of the SEP by the well-known Arnoldi iteration algorithm, implemented in ARPACK routines.

In [7] the same method is used to determine the transversal mode for a thin walled profile, with the difference that they use a 3D Timoshenko beam for their section discretization, thus from the resolution of the SEP they derive the transversal modes and also their corresponding warping mode. We use here a different approach for the determination of the first warping mode for each transversal mode, based on the equilibrium of the beam element in the case of uniform warping; the higher order warping modes will be derived by using an iterative equilibrium scheme.

3. Determination of warping functions modes for a given transversal mode

3.1. The first warping mode determination

We consider the kinematics of a beam element, where we include only one transversal deformation mode. We then write the displacement vector $\mathbf{d}$ of an arbitrary point P of the section:

$$\boldsymbol{d} = \begin{Bmatrix} u_p \\ v_p \\ w_p \end{Bmatrix} = \begin{Bmatrix} u_p \\ \zeta\psi_y \\ \zeta\psi_z \end{Bmatrix} \tag{3}$$

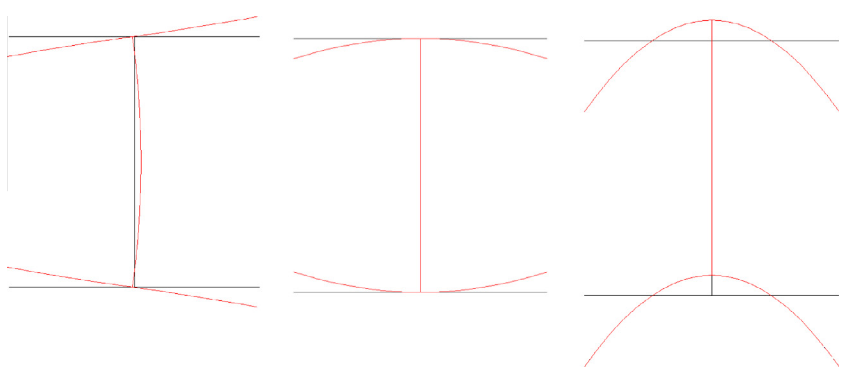

Fig. 1. Examples of transversal deformation modes for a thin walled profile I-section with 1D elements.

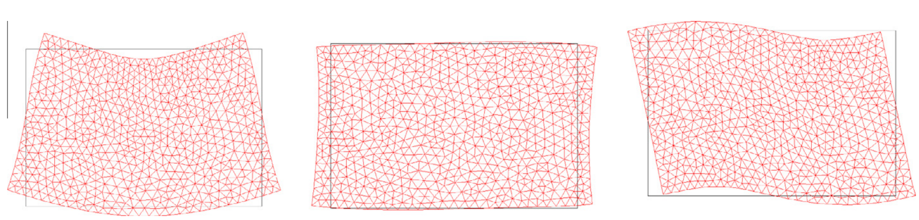

Fig. 2. Examples of transversal deformation modes for a rectangular section with triangular elements.