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Two-point gradient-based MMA (TGMMA) algorithm for topology optimization

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ABSTRACT

Dual methods based on sequential approximations are usually employed for solving topology optimization problems. Among the approximation methods, the method of moving asymptotes (MMA) is perhaps one of the most popular methods used for solving these problems (Svanberg, 1987) [1]. However, recent investigations have shown poor performance of the MMA algorithm as compared to other approximations (Groenwold and Etman, 2010) [2]. In this paper we propose a two-point gradient based MMA approximation, termed as TGMMA, to improve the performance of the MMA algorithm. Numerical results demonstrate the efficiency of the TGMMA algorithm, which improves the MMA algorithm and also shows better performance over other existing approximations.

1. Introduction

Structural topology optimization is a mathematical process of finding the optimal layout of prescribed amount of material within a given domain, with the aim of optimizing desired performance objectives. Topology optimization methods have been applied in variety of applications including structural system design, automobile manufacture, and space vehicle design [3–7], among others. Optimization algorithms based on optimality criteria (OC) methods [8,9] is one of the popular algorithms used in topology optimization. A drawback of the OC methods is that it is not easy to handle multiple constraints [10]. Alternative choice is using dual optimization methods based on sequential approximations, referred to as dual sequential approximation (DSA) methods in this study.

A considerable amount of research has been done on DSA algorithms after Schmit's [11,12] first introduction of the approximation concepts for solving large scale structural optimization problems. The main steps involved in the DSA methods are as follows (Fig. 1) [13]: (a) approximated sub-problem: starting with a feasible design point, generate a convex and separable (local) approximation of the primal objective function and constraints at that design point; (b) dual function: compute the dual of the sub-problem; (c) solve the dual problem; since the objective and constraint functions in the sub-problem are convex, there is not duality gap [14]; (d) update the design point and go to the next iteration until the termination criteria is satisfied. The DSA algorithms are well suited for topology optimization problems where the number of design variables far exceeds the number of constraints, and thus, working in dual spaces results in computational efficiency [15]. It has been recently shown that the well-known OC method for topology optimization can be derived from the DSA methods [10], and therefore, the DSA methods forms a general class of algorithms that can be used for topology optimization.

In the framework of DSA algorithms, the accuracy and efficiency of the optimization scheme depends on the quality of the (approximated) sub-problem. Thus, the formulation of effective and accurate sub-problems is still one of the active areas of research in structural optimization. In early studies, the approximations were constructed using first-order truncated Taylor series expansion (TSE) with intervening variables. For instance, Schmit and Fleury [12] used linear and reciprocal approximations; convex linearization (CONLIN) proposed by Fleury [16,17] employed a combination of linear and reciprocal approximations depending on the gradient signs at current design point. Svanberg [1] later relaxed the conservatism of CONLIN by adding and updating the moving asymptotes at every iteration to create the well-known method of moving asymptotes (MMA). Groenwold [10] used an exponential approximation to develop algorithms for topology optimization. These aforementioned local approximations can be sometimes improved by history information from the previous iterations. To this end, multi-point enhancement approaches are sometimes employed, among which the two-point enhancement

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is widely used. For instance, Haftka et al. [18,19] suggested several two-point methods such as modified reciprocal, two-point projection and an exponential approximation; Fadel et al. [20] proposed a two-point exponential algorithm by matching the gradient of the approximate sub-problem to the exact value at previous design point; Wang and Grandhi [21] and Xu and Grandhi [22] proposed two-point adaptive nonlinear approximation family by matching both of the gradient and function value at the previous design points.

To further improve the approximations, second-order or higher-order TSE can be used. Higher-order TSE is more accurate than the first-order TSE since it retains better curvature information at the design point. However, higher than second-order TSE are seldom used because it is cumbersome to compute and store the higher-order derivatives when solving large-scale problems. Fleury [23] showed that replacing the fully populated Hessian by diagonal Hessian approximation may also improve the performance when using sequential quadratic programming (SQP) for structural optimization problems. However, for topology optimization problems the Hessian is approximated using the information from the first order derivatives at the design point as it is difficult to calculate the Hessian directly. In order to improve the performance of second order approximations, Groenwold [24] recently proposed an incomplete series expansion (ISE) family of algorithms that are based on the diagonal approximation of the Hessian using reciprocal and exponential approximations.

In order to use the DSA framework, the sub-problem should be both convex and separable so that the dual function can be easily computed [15]. The MMA algorithm constructs sub-problem that satisfies these criteria, and is one of the most popular methods in topology optimization. However, inferior performance of the MMA algorithm has been reported for topology optimization problems when compared to other recently proposed algorithms based on reciprocal, exponential and second order approximations [2,10]. In this paper, the performance of the MMA algorithm on topology optimization problem is investigated, and enhancements to the MMA algorithm for topology optimization are proposed using the two-point gradient enforcement [20]. The improved algorithm is referred to as two-point gradient based MMA (TGMMA). The TGMMA provides a better local approximation than the MMA, and is shown to alleviate the oscillations issues that existed in the MMA algorithm. The performance of TGMMA is compared with the other algorithms including reciprocal, two-point exponential, MMA and second order methods. Optimal minimum compliance and optimal compliant mechanism design problems are presented to demonstrate the efficiency of the proposed TGMMA algorithm. The outline of this paper is as follows: In Section 2 the framework of DSA algorithm is presented. Section 3 describes the different approximations used in topology optimization. The proposed TGMMA algorithm is explained in Section 4, and performance evaluation of various algorithms is carried out in Section 5. Finally, the important conclusions and remarks are given in Section 6.

2. Dual sequential approximation algorithm

2.1. Primal problem

A typical topology optimization problem with inequality constraints can be expressed as follows:

$$\begin{aligned} &\min_{x\in\mathcal{B}} f_0(\boldsymbol{x}) \\ &\text{Subject to :} \\ &\boldsymbol{g}(\boldsymbol{x}) \leqslant \boldsymbol{0} \\ &\boldsymbol{x} \in \mathcal{B} = \{\boldsymbol{x} | \boldsymbol{x}_l \leqslant \boldsymbol{x} \leqslant \boldsymbol{x}_u\} \subset \mathbb{R}^n \end{aligned} \tag{1}$$

where $\boldsymbol{x} \in \mathbb{R}^n$ is the design variable; $f_0(\boldsymbol{x}) : \mathbb{R}^n \to \mathbb{R}$ is the objective function (compliance or any other quantity of interest) that needs to be minimized; $\boldsymbol{g}(\boldsymbol{x}) : \mathbb{R}^n \to \mathbb{R}^m$ are the m inequality constraints; and $\boldsymbol{x}_l$ and $\boldsymbol{x}_u$ in the box constraints (Eq. (1_3)) define the lower and upper boundary of the design variables, respectively.

2.2. Convex and separable primal sub-problem

In topology optimization problems, the objective function, the constraints and their derivatives are obtained by numerical methods (e.g. finite element analysis) and their explicit expressions are usually not available. Furthermore, in most optimization problems, the objective and constraint functions are neither convex nor separable, and therefore, direct optimization methods are inefficient for these problems [25]. Alternatively, DSA algorithms start with the construction of convex and separable approximations of the objective and constraint functions at the current design point x^k, and the primal problem is replaced with a sequence of approximate sub-problems. The approximated sub-problem is expressed as follows:

$$\begin{aligned} &\min_{\boldsymbol{x}\in\mathcal{B}} \hat{f}_0^k(\boldsymbol{x}) \\ &\text{Subject to :} \\ &\hat{\boldsymbol{g}}^k(\boldsymbol{x}) \leqslant \boldsymbol{0} \\ &\boldsymbol{x} \in \mathcal{B} = \{\boldsymbol{x} | \boldsymbol{x}_l \leqslant \boldsymbol{x} \leqslant \boldsymbol{x}_u\} \end{aligned} \tag{2}$$

where $\hat{f}_0^k(\boldsymbol{x}) : \mathbb{R}^n \to \mathbb{R}$ and $\hat{\boldsymbol{g}}^k(x) : \mathbb{R}^n \to \mathbb{R}^m$ are the convex and separable approximations of the primal objective function and constraints, respectively, at the current design point $\boldsymbol{x}^k$. In the sub-problem, we start with an initial guess $\boldsymbol{x}^k$, which is the optimal design point of the last iteration. Defining as the dual optimizer, consider the following mapping:

$$\hat{\boldsymbol{x}}^{k+1} = \mathcal{D}(\boldsymbol{x}^k) \tag{3}$$

where the optimal solution $\hat{\boldsymbol{x}}^{k+1}$ of the sub-problem (Eq. (2)) can be seen as the mapping $\mathcal{D} : \boldsymbol{x}^k \to \hat{\boldsymbol{x}}^{k+1}$. With this mapping, the change between optimized solution and initial input is:

$$\Delta\boldsymbol{x}^k = \hat{\boldsymbol{x}}^{k+1} - \boldsymbol{x}^k \tag{4}$$

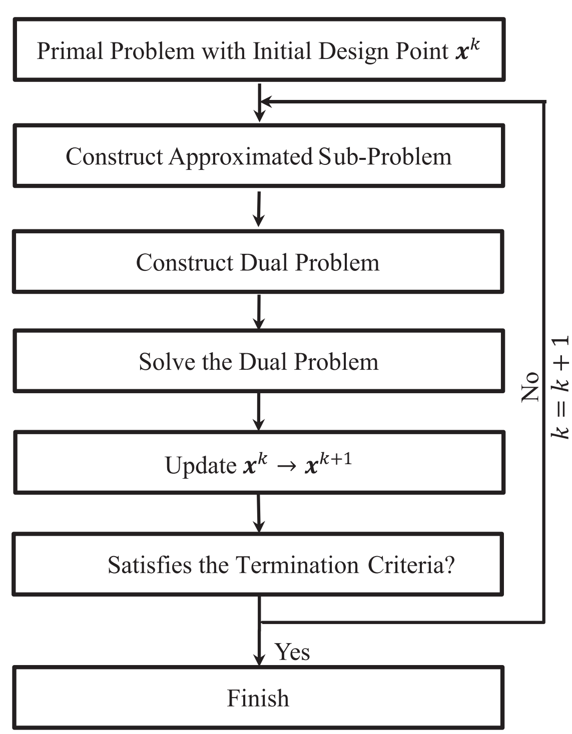

Fig. 1. DSA algorithm for solving topology optimization problem.