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On the computation of dispersion curves for axisymmetric elastic waveguides using the Scaled Boundary Finite Element Method

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ABSTRACT

In this paper we propose an algorithm to compute specific parts of the dispersion curves for elastic waveguides. The formulation is based on an axisymmetric representation of the Scaled Boundary Finite Element Method, where the wavenumbers of propagating modes are obtained as solutions of a Hamiltonian eigenvalue problem. The novel solution procedure involves tracing selected modes over a given frequency range and computing the corresponding solutions by means of inverse iteration. The resulting algorithm is applied in the context of material characterization, where the efficiency of the computation is crucial.

1. Introduction

Ultrasonic guided waves offer a variety of applications in fields such as non-destructive testing [1–3], structural health monitoring [4–6] or material characterization [7–10]. Particularly, cylindrical waveguides are of interest in many engineering applications. Due to the complex dispersive behavior of guided waves, accurate modeling of the propagating and/or evanescent modes is required. Numerous analytical and numerical approaches have been developed over the last decades, in order to compute dispersion curves and mode shapes for waveguides with different geometries and varying distribution of material parameters. For instance, the well-known Global Matrix Method [11–13], which uses the Pochhammer–Chree theory [14,15], is based on the analytical description of the partial waves being transmitted and reflected at the waveguide's surfaces. Most numerical approaches rest on the concept of Finite Elements, by discretizing a representative part of the waveguide [16] or the cross-section only. The latter approach, often referred to as Semi-Analytical Finite Element (SAFE) method, has been applied to a variety of waveguides with different geometries, boundary conditions and distribution of material parameters, see e.g., [17–19]. Recently, it has been demonstrated that the concept of the Scaled Boundary Finite Element Method (SBFEM) can be employed to compute dispersion curves for arbitrary waveguides very efficiently [20–22]. The SBFEM is generally a semi-analytical method [23–25]. The boundary of the domain under consideration is discretized in the Finite Element sense while analytical formulations are derived for the interior of the domain. This concept has been applied successfully for solving different problems in time and frequency domain. Among other applications this method has shown to be advantageous for the simulation of elastic waves in bounded [26,27] and unbounded domains [28–30]. Employing the SBFEM for the computation of dispersion curves for guided waves leads to the cross-section of the waveguide to be discretized, similar to the SAFE method. The solution procedures described in [20–22] allow for a highly efficient computation of wavenumbers, mode shapes and group velocities of propagating modes in the waveguide. A standard eigenvalue problem is solved at each frequency of interest. Utilizing spectral elements of very high order [31] has shown to drastically improve efficiency in comparison with traditional quadratic elements.

However, in many applications it is worthwhile to further improve efficiency by considering the specific requirements of the set-up. In the current contribution, we focus on an algorithm developed for a measurement system that utilizes guided waves for material characterization [10,32–34]. In this application, dispersion curves have to be computed for a large frequency range. As the computation has to be performed many times with varying material parameters, the efficiency of the algorithm is crucial. The approach is based on an axisymmetric formulation of the SBFEM as described in previous work [20,21]. In this paper, a novel solution procedure is proposed in order to significantly increase its computational efficiency. Similar to many other applications of guided

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waves, only few modes can actually be excited at each frequency, due to the geometrical characteristics of the excitation. In the proposed approach, only the modes of interest are computed, rather than solving for the complete set of solutions at each frequency. This is done by tracing the selected modes [35] and computing the required solutions using the concept of inverse iteration [36,37]. In the current application we focus on homogeneous axisymmetric waveguides with isotropic or orthotropic material behavior. However, the proposed algorithm can easily be adopted to include more general waveguides.

The structure of the paper is as follows. In Section 2.1 we briefly introduce the experimental set-up that motivates the development of the novel solution procedure. Section 2.2 summarizes the governing equations and the SBFEM formulation as far as it is required to explain the following steps. The solution procedure is developed in Section 3. In Section 4 some aspects of the implementation are described in more detail. Numerical examples are presented in Section 5, also demonstrating efficiency and convergence behavior of the proposed approach. A conclusion is given in Section 6.

2. Background

2.1. Motivation

The proposed algorithm is utilized in the context of ultrasonic material characterization of cylindrical waveguides as presented in [10,34,38]. The experimental set-up (Fig. 1a) consists of an ultrasonic transmitter, a hollow cylindrical waveguide and an ultrasonic receiver. Measurements are performed in transmission between the parallel faces. Fig. 1b shows an example for the pulse excited by the transmitter and the resulting signal at the receiver. A plane-wave approximation yields a first estimation of the material parameters. The estimated values are used to initialize an inverse approach. Using an optimization algorithm, the material parameters of the waveguide are modified until the simulated and measured signals are consistent. As the simulation of the waveguide's dispersive behavior has to be performed in every iteration of the optimization process, an efficient and numerically stable waveguide model is desired. To increase efficiency, we first have a closer look at the required waveguide modes to be computed. Due to the spatially homogeneous excitation on the cylinder's cross-section, only longitudinal modes are propagating through the sample. If we model a plane excitation of normal tractions on the cylinder's cross-section, e.g., using the reciprocity theorem [39,40] or a least-squares approach [10,34], we obtain the modal amplitude of each mode at a given frequency. Fig. 2 shows typical results for a polymeric waveguide consisting of natural polypropylene (PPN) with a longitudinal wave velocity of $c_l = 2.7$ km/s and a Possion's ratio of $\nu = 0.35$. The inner and outer radius is chosen as 3 mm and 9 mm, respectively. Due to the symmetric and normal excitation, only modes

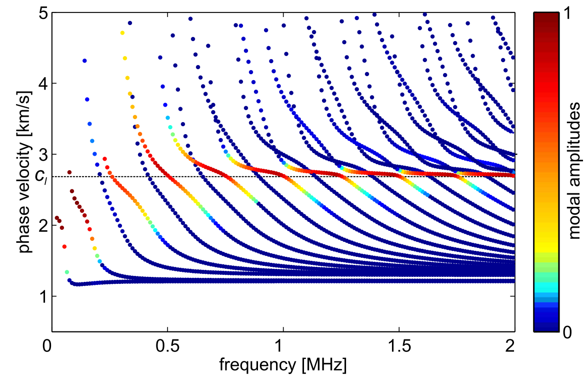


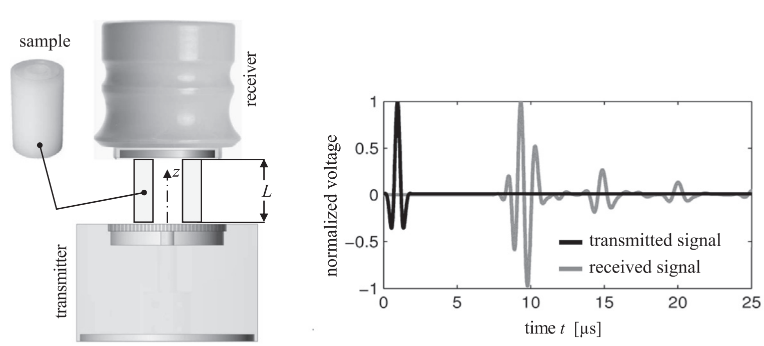
Fig. 2. Normalized modal amplitudes of the longitudinal modes in a hollow PPN cylinder.

with a phase velocity close to c_l will propagate in the waveguide. Additionally, the spectral range of the transmitter defines the frequency range of interest. It is interesting to note that (except for the fundamental Mode L(0,0) at low frequencies) only modes with a symmetric mode shape can be excited, as the modeled excitation itself is symmetric. This prior knowledge of the waveguide modes that can be excited using the given experimental set-up, will be utilized to develop an efficient algorithm for computing the dispersive behavior of a cylindrical waveguide.

2.2. SBFEM for cylindrical waveguides

In previous work, SBFEM formulations have been presented to describe waveguides of different geometry and arbitrary distribution of material parameters [20–22]. In the current paper we focus on homogeneous axisymmetric waveguides. Only the equations required to develop the novel solution procedure are summarized in this subsection. The hollow cylinder depicted in Fig. 3 is addressed. The geometry is described in a cylindrical coordinate system $(z,\ \theta,\ r)$. The inner radius and the thickness are denoted by r_i and l, respectively. To analyze the modes that can propagate in the waveguide, we can assume the structure to be of infinite length. In terms of the SBFEM, the discretization of the cross-section is scaled between $z = 0$ and $z = \infty$ with respect to a scaling center placed at $z = -\infty$. Since in the current application, the geometry as well as the displacement field are axisymmetric, only the through-thickness direction of the waveguide is discretized. Fig. 4 shows an example for the discretization using one line element. The element is defined in the local coordinate η, which equals −1 and 1 at the element's extremities, respectively.

In the current application we are only interested in longitudinal modes, i.e., two degrees of freedom are assigned to each node of the discretization and the displacements and stresses are



(a) (b)

Fig. 1. (a) Experimental set-up for the material characterization of cylindrical waveguides and (b) example for a measured signal [34].